

**BINOMIAL THEOREM**

1.  ${}^{n+1}C_r = (k^2 - 8) {}^nC_{r+1}$  if and only if :  
 (1)  $2\sqrt{2} < k \leq 3$                       (2)  $2\sqrt{3} < k \leq 3\sqrt{2}$   
 (3)  $2\sqrt{3} < k < 3\sqrt{3}$                 (4)  $2\sqrt{2} < k < 2\sqrt{3}$
2. If A denotes the sum of all the coefficients in the expansion of  $(1 - 3x + 10x^2)^n$  and B denotes the sum of all the coefficients in the expansion of  $(1 + x^2)^n$ , then :  
 (1)  $A = B^3$                                       (2)  $3A = B$   
 (3)  $B = A^3$                                       (4)  $A = 3B$
3. The coefficient of  $x^{2012}$  in the expansion of  $(1-x)^{2008}(1+x+x^2)^{2007}$  is equal to
4. If  $\frac{{}^{11}C_1}{2} + \frac{{}^{11}C_2}{3} + \dots + \frac{{}^{11}C_9}{10} = \frac{n}{m}$  with  $\text{gcd}(n, m) = 1$ , then  $n + m$  is equal to
5. Remainder when  $64^{32^{32}}$  is divided by 9 is equal to \_\_\_\_.
6. Number of integral terms in the expansion of  $\left\{ 7^{\left(\frac{1}{2}\right)} + 11^{\left(\frac{1}{6}\right)} \right\}^{824}$  is equal to \_\_\_\_\_.
7. Suppose  $2 - p, p, 2 - \alpha, \alpha$  are the coefficient of four consecutive terms in the expansion of  $(1+x)^n$ . Then the value of  $p^2 - \alpha^2 + 6\alpha + 2p$  equals  
 (1) 4    (2) 10  
 (3) 8    (4) 6
8. Let  $\alpha = \sum_{k=0}^n \left( \frac{{}^nC_k}{k+1} \right)^2$  and  $\beta = \sum_{k=0}^{n+1} \left( \frac{{}^nC_k \cdot {}^nC_{k+1}}{k+2} \right)$ .  
 If  $5\alpha = 6\beta$ , then  $n$  equals \_\_\_\_\_.
9. In the expansion of  $(1+x)(1-x^2) \left( 1 + \frac{3}{x} + \frac{3}{x^2} + \frac{1}{x^3} \right)^5$ ,  $x \neq 0$ , the sum of the coefficient of  $x^3$  and  $x^{-13}$  is equal to \_\_\_\_
10. Let the coefficient of  $x^r$  in the expansion of  $(x+3)^{n+1} + (x+3)^{n-2}(x+2) + (x+3)^{n-3}(x+2)^2 + \dots + (x+2)^{n+1}$  be  $\alpha_r$ . If  $\sum_{r=0}^n \alpha_r = \beta^n - \gamma^n, \beta, \gamma \in \mathbb{N}$ , then the value of  $\beta^2 + \gamma^2$  equals \_\_\_\_\_.

11. If the Coefficient of  $x^{30}$  in the expansion of  $\left( 1 + \frac{1}{x} \right)^6 (1+x^2)^7 (1-x^3)^8$ ;  $x \neq 0$  is  $\alpha$ , then  $|\alpha|$  equals \_\_\_\_\_.
12. Let  $m$  and  $n$  be the coefficients of seventh and thirteenth terms respectively in the expansion of  $\left( \frac{1}{3}x^{\frac{1}{3}} + \frac{1}{2x^{\frac{2}{3}}} \right)^{18}$ . Then  $\left( \frac{n}{m} \right)^{\frac{1}{3}}$  is :  
 (1)  $\frac{4}{9}$     (2)  $\frac{1}{9}$   
 (3)  $\frac{1}{4}$     (4)  $\frac{9}{4}$
13. If the term independent of  $x$  in the expansion of  $\left( \sqrt{ax^2} + \frac{1}{2x^3} \right)^{10}$  is 105, then  $a^2$  is equal to :  
 (1) 4    (2) 9  
 (3) 6    (4) 2
14. The sum of the coefficient of  $x^{2/3}$  and  $x^{-2/5}$  in the binomial expansion of  $\left( x^{2/3} + \frac{1}{2}x^{-2/5} \right)^9$  is :  
 (1) 21/4    (2) 69/16  
 (3) 63/16    (4) 19/4
15. The coefficient of  $x^{70}$  in  $x^2(1+x)^{98} + x^3(1+x)^{97} + x^4(1+x)^{96} + \dots + x^{54}(1+x)^{46}$  is  ${}^{99}C_p - {}^{46}C_q$ . Then a possible value to  $p + q$  is :  
 (1) 55    (2) 61  
 (3) 68    (4) 83
16. The remainder when  $428^{2024}$  is divided by 21 is \_\_\_\_\_.
17. The sum of all rational terms in the expansion of  $\left( 2^{\frac{1}{5}} + 5^{\frac{1}{3}} \right)^{15}$  is equal to :  
 (1) 3133    (2) 633  
 (3) 931    (4) 6131

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18. Let  $a = 1 + \frac{{}^2C_2}{{}^3!} + \frac{{}^3C_2}{{}^4!} + \frac{{}^4C_2}{{}^5!} + \dots$ ,  
 $b = 1 + \frac{{}^1C_0 + {}^1C_1}{{}^1!} + \frac{{}^2C_0 + {}^2C_1 + {}^2C_2}{{}^2!} + \frac{{}^3C_0 + {}^3C_1 + {}^3C_2 + {}^3C_3}{{}^3!} + \dots$
- Then  $\frac{2b}{a^2}$  is equal to \_\_\_\_\_
19. If the coefficients of  $x^4$ ,  $x^5$  and  $x^6$  in the expansion of  $(1 + x)^n$  are in the arithmetic progression, then the maximum value of  $n$  is :
- (1) 14 (2) 21  
 (3) 28 (4) 7
20. If the constant term in the expansion of  $\left(\frac{\sqrt[5]{3}}{x} + \frac{2x}{\sqrt[3]{5}}\right)^{12}$ ,  $x \neq 0$ , is  $\alpha \times 2^8 \times \sqrt[5]{3}$ , then  $25\alpha$  is equal to :
- (1) 639 (2) 724  
 (3) 693 (4) 742

21. Let  $0 \leq r \leq n$ . If  ${}^{n+1}C_{r+1} : {}^nC_r : {}^{n-1}C_{r-1} = 55 : 35 : 21$ , then  $2n + 5r$  is equal to:
- (1) 60 (2) 62  
 (3) 50 (4) 55
22. If the constant term in the expansion of  $(1 + 2x - 3x^2)\left(\frac{3}{2}x^2 - \frac{1}{3x}\right)^9$  is  $p$ , then  $108p$  is equal to
23. If the second, third and fourth terms in the expansion of  $(x + y)^n$  are  $135$ ,  $30$  and  $\frac{10}{3}$ , respectively, then  $6(n^3 + x^2 + y)$  is equal to \_\_\_\_\_.
24. Let  $\alpha = \sum_{r=0}^n (4r^2 + 2r + 1) {}^nC_r$   
 and  $\beta = \left(\sum_{r=0}^n \frac{{}^nC_r}{r+1}\right) + \frac{1}{n+1}$ . If  $140 < \frac{2\alpha}{\beta} < 281$ , then the value of  $n$  is \_\_\_\_\_.

**SOLUTIONS**

**1. Ans. (1)**

**Sol.**  ${}^{n-1}C_r = (k^2 - 8) {}^nC_{r+1}$

$$r + 1 \geq 0, r \geq 0$$

$$\frac{{}^{n-1}C_r}{{}^nC_{r+1}} = k^2 - 8$$

$$\frac{r+1}{n} = k^2 - 8$$

$$\Rightarrow k^2 - 8 > 0$$

$$(k - 2\sqrt{2})(k + 2\sqrt{2}) > 0$$

$$k \in (-\infty, -2\sqrt{2}) \cup (2\sqrt{2}, \infty) \quad \dots(I)$$

$$\therefore n \geq r + 1, \frac{r+1}{n} \leq 1$$

$$\Rightarrow k^2 - 8 \leq 1$$

$$k^2 - 9 \leq 0$$

$$-3 \leq k \leq 3 \quad \dots(II)$$

From equation (I) and (II) we get

$$k \in [-3, -2\sqrt{2}) \cup (2\sqrt{2}, 3]$$

**2. Ans. (1)**

**Sol.** Sum of coefficients in the expansion of

$$(1 - 3x + 10x^2)^n = A$$

$$\text{then } A = (1 - 3 + 10)^n = 8^n \text{ (put } x = 1)$$

and sum of coefficients in the expansion of

$$(1 + x^2)^n = B$$

$$\text{then } B = (1 + 1)^n = 2^n$$

$$A = B^3$$

**3. Ans. (0)**

**Sol.**  $(1-x)(1-x)^{2007}(1+x+x^2)^{2007}$

$$(1-x)(1-x^3)^{2007}$$

$$(1-x)({}^{2007}C_0 - {}^{2007}C_1(x^3) + \dots)$$

General term

$$(1-x)((-1)^r {}^{2007}C_r x^{3r})$$

$$(-1)^r {}^{2007}C_r x^{3r} - (-1)^{r+1} {}^{2007}C_{r+1} x^{3r+1}$$

$$3r = 2012$$

$$r \neq \frac{2012}{3}$$

$$3r + 1 = 2012$$

$$3r = 2011$$

$$r \neq \frac{2011}{3}$$

Hence there is no term containing  $x^{2012}$ .

So coefficient of  $x^{2012} = 0$

**4. Ans. (2041)**

**Sol.**  $\sum_{r=1}^9 \frac{{}^{11}C_r}{r+1}$

$$= \frac{1}{12} \sum_{r=1}^9 {}^{12}C_{r+1}$$

$$= \frac{1}{12} [2^{12} - 26] = \frac{2035}{6}$$

$$\therefore m + n = 2041$$

**5. Ans. (1)**

**Sol.** Let  $32^{32} = t$

$$64^{32^{32}} = 64^t = 8^{2t} = (9-1)^{2t}$$

$$= 9k + 1$$

Hence remainder = 1

**6. Ans. (138)**

**Sol.** General term in expansion of  $(7)^{1/2} + (11)^{1/6}$  is

$$t_{r+1} = {}^{824}C_r (7)^{\frac{824-r}{2}} (11)^{r/6}$$

For integral term, r must be multiple of 6.

Hence  $r = 0, 6, 12, \dots, 822$

**7. Ans. (Bonus)**

**Sol.**  $2 - p, p, 2 - \alpha, \alpha$

Binomial coefficients are

${}^nC_r, {}^nC_{r+1}, {}^nC_{r+2}, {}^nC_{r+3}$  respectively

$$\Rightarrow {}^nC_r + {}^nC_{r+1} = 2$$

$$\Rightarrow {}^{n+1}C_{r+1} = 2 \quad \dots(1)$$

Also,  ${}^nC_{r+2} + {}^nC_{r+3} = 2$

$$\Rightarrow {}^{n+1}C_{r+3} = 2 \quad \dots(2)$$

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From (1) and (2)

$${}^{n+1}C_{r+1} = {}^{n+1}C_{r+3}$$

$$\Rightarrow 2r + 4 = n + 1$$

$$n = 2r + 3$$

$${}^{2r+4}C_{r+1} = 2$$

Data Inconsistent

**8. Ans. (10)**

**Sol.** 
$$\alpha = \sum_{k=0}^n \frac{{}^nC_k \cdot {}^nC_k}{k+1} \cdot \frac{n+1}{n+1}$$

$$= \frac{1}{n+1} \sum_{k=0}^n {}^{n+1}C_{k+1} \cdot {}^nC_{n-k}$$

$$\alpha = \frac{1}{n+1} \cdot {}^{2n+1}C_{n+1}$$

$$\beta = \sum_{k=0}^{n+1} {}^nC_k \cdot \frac{{}^nC_{k+1} \cdot n+1}{k+2}$$

$$\frac{1}{n+1} \sum_{k=0}^{n+1} {}^nC_{n-k} \cdot {}^{n+1}C_{k+2}$$

$$= \frac{1}{n+1} \cdot {}^{2n+1}C_{n+2}$$

$$\frac{\beta}{\alpha} = \frac{{}^{2n+1}C_{n+2}}{{}^{2n+1}C_{n+1}} = \frac{2n+1 - (n+2) + 1}{n+2}$$

$$\frac{\beta}{\alpha} = \frac{n}{n+2} = \frac{5}{6}$$

$$n = 10$$

**9. Ans. (118)**

**Sol.** 
$$(1+x)(1-x^2) \left( 1 + \frac{3}{x} + \frac{3}{x^2} + \frac{1}{x^3} \right)^5$$

$$= (1+x)(1-x^2) \left( \left( 1 + \frac{1}{x} \right)^3 \right)^5$$

$$= \frac{(1+x)^2 (1-x) (1+x)^{15}}{x^{15}}$$

$$= \frac{(1+x)^{17} - x(1+x)^{17}}{x^{15}}$$

= coeff( $x^3$ ) in the expansion  $\approx$ coeff( $x^{18}$ ) in

$$(1+x)^{17} - x(1+x)^{17}$$

$$= 0 - 1$$

$$= -1$$

coeff( $x^{-13}$ ) in the expansion  $\approx$ coeff( $x^2$ ) in  $(1+x)^{17} - x(1+x)^{17}$

$$= \binom{17}{2} - \binom{17}{1}$$

$$= 17 \times 8 - 17$$

$$= 17 \times 7$$

$$= 119$$

Hence Answer = 119 - 1 = 118

**10. Ans. (25)**

**Sol.** 
$$(x+3)^{n+1} + (x+3)^{n-2} (x+2) + (x+3)^{n-3}$$

$$(x+2)^2 + \dots + (x+2)^{n+1}$$

$$\sum \alpha_r = 4^{n+1} + 4^{n-2} \cdot 3 + 4^{n-3} \cdot 3^2 \dots + 3^{n+1}$$

$$= 4^{n+1} \left[ 1 + \frac{3}{4} + \left( \frac{3}{4} \right)^2 \dots + \left( \frac{3}{4} \right)^{n+1} \right]$$

$$= 4^{n+1} \cdot \frac{1 - \left( \frac{3}{4} \right)^{n+1}}{1 - \frac{3}{4}}$$

$$= 4^n - 3^n = \beta^n - \gamma^n$$

$$\beta = 4, \gamma = 3$$

$$\beta^2 + \gamma^2 = 16 + 9 = 25$$

**11. Ans. (678)**

**Sol.** coeff of  $x^{30}$  in  $\frac{(x+1)^6 (1+x^2)^7 (1-x^3)^8}{x^6}$

coeff. of  $x^{36}$  in  $(1+x)^6 (1+x^2)^7 (1-x^3)^8$

General term

$${}^6C_{r_1} {}^7C_{r_2} {}^8C_{r_3} (-1)^{r_3} x^{r_1+2r_2+3r_3}$$

$$r_1 + 2r_2 + 3r_3 = 36$$

$r_1$	$r_2$	$r_3$
0	6	8
2	5	8
4	4	8
6	3	8

Case-I:  $r_1 + 2r_2 = 12$  (Taking  $r_3 = 8$ )

r <sub>1</sub>	r <sub>2</sub>	r <sub>3</sub>
1	7	7
3	6	7
5	5	7

Case-II :  $r_1 + 2r_2 = 15$  (Taking  $r_3 = 7$ )

r <sub>1</sub>	r <sub>2</sub>	r <sub>3</sub>
4	7	6
6	6	6

Case-III :  $r_1 + 2r_2 = 18$  (Taking  $r_3 = 6$ )

Coeff. =  $7 + (15 \times 21) + (15 \times 35) + (35) - (6 \times 8) - (20 \times 7 \times 8) - (6 \times 21 \times 8) + (15 \times 28) + (7 \times 28)$   
 $= -678 = \alpha$

$|\alpha| = 678$

12. Ans. (4)

Sol.  $\left(\frac{x^{\frac{1}{3}}}{3} + 2x^{\frac{2}{3}}\right)^{18}$

$t_7 = {}^{18}C_6 \left(\frac{x^{\frac{1}{3}}}{3}\right)^{12} \left(x^{\frac{2}{3}}\right)^6 = {}^{18}C_6 \frac{1}{(3)^{12}} \cdot \frac{1}{2^6}$

$t_{13} = {}^{18}C_{12} \left(\frac{x^{\frac{1}{3}}}{3}\right)^6 \left(x^{\frac{2}{3}}\right)^{12} = {}^{18}C_{12} \frac{1}{(3)^6} \cdot \frac{1}{2^{12}} \cdot x^{-6}$

$m = {}^{18}C_6 \cdot 3^{-12} \cdot 2^{-6} : n = {}^{18}C_{12} \cdot 2^{-12} \cdot 3^{-6}$

$\left(\frac{n}{m}\right)^{\frac{1}{3}} = \left(\frac{2^{-12} \cdot 3^{-6}}{3^{-12} \cdot 2^{-6}}\right)^{\frac{1}{3}} = \left(\frac{3}{2}\right)^2 = \frac{9}{4}$

13. Ans. (1)

Sol.  $\left(\sqrt{ax^2} + \frac{1}{2x^3}\right)^{10}$

General term =  ${}^{10}C_r (\sqrt{ax^2})^{10-r} \left(\frac{1}{2x^3}\right)^r$

$20 - 2r - 3r = 0$

$r = 4$

${}^{10}C_4 a^3 \cdot \frac{1}{16} = 105$

$a^3 = 8$

$a^2 = 4$

14. Ans. (1)

Sol.  $T_{r+1} = {}^9C_r (x^{2/3})^{9-r} \left(\frac{x^{-2/5}}{2}\right)^r$   
 $= {}^9C_r \left(\frac{1}{2}\right)^r (r)^{\binom{6 \cdot \frac{2r}{3} - \frac{2r}{5}}{}}$

for coefficient of  $x^{2/3}$ , put  $6 - \frac{2r}{3} - \frac{2r}{5} = \frac{2}{3}$

$\Rightarrow r = 5$

$\therefore$  Coefficient of  $x^{2/3}$  is  ${}^9C_5 \left(\frac{1}{5}\right)^5$

For coefficient of  $x^{-2/5}$ , put  $6 - \frac{2r}{3} - \frac{2r}{5} = -\frac{2}{5}$

$\Rightarrow r = 6$

Coefficient of  $x^{-2/5}$  is  ${}^9C_6 \left(\frac{1}{2}\right)^6$

Sum =  ${}^9C_5 \left(\frac{1}{2}\right)^5 + {}^9C_6 \left(\frac{1}{2}\right)^6 = \frac{21}{4}$

15. Ans. (4)

Sol.  $x^2(1+x)^{98} + x^3(1+x^{97}) + x^4(1+x)^{96} + \dots$   
 $x^{54}(1+x)^{46}$

Coeff. of  $x^{70} : {}^{98}C_{68} + {}^{97}C_{67} + {}^{96}C_{66} + \dots$

${}^{47}C_{17} + {}^{46}C_{16}$   
 $= {}^{46}C_{30} + {}^{47}C_{30} + \dots + {}^{98}C_{30}$   
 $= ({}^{46}C_{31} + {}^{46}C_{30}) + {}^{47}C_{30} + \dots + {}^{98}C_{30} - {}^{46}C_{31}$   
 $= {}^{47}C_{31} + {}^{47}C_{30} + \dots + {}^{98}C_{30} - {}^{46}C_{31} \dots$   
 $= {}^{99}C_{31} - {}^{46}C_{31} = {}^{99}C_p - {}^{46}C_q$

Possible values of  $(p + q)$  are 62, 83, 99, 46

$\Rightarrow p + q = 83$

16. Ans. (1)

Sol.  $(428)^{2024} = (420 + 8)^{2024}$

$= (21 \times 20 + 8)^{2024}$

$= 21m + 8^{2024}$

Now  $8^{2024} = (8^2)^{1012}$

$= (64)^{1012}$

$= (63 + 1)^{1012}$

$= (21 \times 3 + 1)^{1012}$

$= 21n + 1$

$\Rightarrow$  Remainder is 1.

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**17. Ans. (1)**

**Sol.**  $T_{r+1} = {}^{15}C_r \left(5^{\frac{1}{3}}\right)^r \left(2^{\frac{1}{5}}\right)^{15-r}$   
 $= {}^{15}C_r 5^{\frac{r}{3}} \cdot 2^{\frac{15-r}{5}}$   
 $R = 3\lambda, 15\mu$   
 $\Rightarrow r = 0, 15$   
 2 rational terms  
 $\Rightarrow {}^{15}C_0 2^3 + {}^{15}C_{15} (5)^5$   
 $= 8 + 3125 = 3133$

**18. Ans. (8)**

**Sol.**  $f(x) = 1 + \frac{(1+x)}{1!} + \frac{(1+x)^2}{2!} + \frac{(1+x)^3}{3!} + \dots$   
 $\frac{e^{(1+x)}}{1+x} = \frac{1}{1+x} + 1 + \frac{(1+x)}{2!} + \frac{(1+x)^2}{3!} + \frac{(1+x)^3}{4!} + \dots$   
 coef  $x^2$  in RHS :  $1 + \frac{{}^2C_2}{3} + \frac{{}^3C_2}{4} + \dots = a$   
 coeff.  $x^2$  in L.H.S.  
 $e \left(1 + x + \frac{x^2}{2!}\right) \dots \left(1 - x + \frac{x^2}{2!} + \dots\right)$   
 is  $e - e + \frac{e}{2!} = a$   
 $b = 1 + \frac{2}{1!} + \frac{2^2}{2!} + \frac{2^3}{3!} + \dots = e^2$   
 $\frac{2b}{a^2} = 8$

**19. Ans. (1)**

**Sol.** Coeff. of  $x^4 = {}^nC_4$   
 Coeff. of  $x^5 = {}^nC_5$   
 Coeff. of  $x^6 = {}^nC_6$   
 ${}^nC_4, {}^nC_5, {}^nC_6 \dots$  AP  
 $2 \cdot {}^nC_5 = {}^nC_4 + {}^nC_6$   
 $2 = \frac{{}^nC_4}{{}^nC_5} + \frac{{}^nC_6}{{}^nC_5} \left\{ \frac{{}^nC_r}{{}^nC_{r-1}} = \frac{n-r+1}{r} \right\}$   
 $2 = \frac{5}{n-4} + \frac{n-5}{6}$   
 $12(n-4) = 30 + n^2 - 9n + 20$   
 $n^2 - 21n + 98 = 0$   
 $(n-14)(n-7) = 0$   
 $n_{\max} = 14 \quad n_{\min} = 7$

**20. Ans. (3)**

**Sol.**  $T_{r+1} = {}^{12}C_r \left(\frac{3^{1/5}}{x}\right)^{12-r} \left(\frac{2x}{5^{1/3}}\right)^r$   
 $T_{r+1} = \frac{{}^{12}C_r (3)^{\frac{12-r}{5}} (2)^r (x)^{2r-12}}{(5)^{\frac{r}{3}}}$   
 $r = 6$   
 $T_7 = \frac{{}^{12}C_6 (3)^{6/5} (2)^6}{5^2} = \left(\frac{9 \cdot 11 \cdot 7}{25}\right) 2^8 \cdot 3^{1/5}$   
 $25\alpha = 693$

**21. Ans. (3)**

**Sol.**  $\frac{{}^{n+1}C_r}{{}^nC_r} = \frac{55}{35}$   
 $\frac{(n+1)!}{(r+1)!(n-r)!} \cdot \frac{r!(n-r)!}{n!} = \frac{11}{7}$   
 $\frac{(n+1)}{r+1} = \frac{11}{7}$   
 $7n = 4 + 11r$   
 $\frac{{}^nC_r}{{}^{n-1}C_{r-1}} = \frac{35}{21}$   
 $\frac{n!}{r!(n-r)!} = \frac{(r-1)!(n-r)!}{(n-1)!} = \frac{5}{3}$   
 $\frac{n}{r} = \frac{5}{3}$   
 $3n = 5r$   
 By solving  $r = 6 \quad n = 10$   
 $2n + 5r = 50$

22. Ans. (54)

Sol.  $(1 + 2x - 3x^3) \left( \frac{3}{2}x^2 - \frac{1}{3x} \right)^9$

General term  $m \left( \frac{3}{2}x^2 - \frac{1}{3x} \right)^9$

$$= {}^9C_r \cdot \frac{3^{9-2r}}{2^{9-r}} (-1)^r \cdot x^{18-3r}$$

Put  $r = 6$  to get coeff. of  $x^0 = {}^9C_6 \cdot \frac{1}{6^3} \cdot x^0 = \frac{7}{18}x^0$

Put  $r = 7$  to get coeff. of  $x^{-3} = {}^9C_7 \cdot \frac{3^{-5}}{2^2} (-1)^7 \cdot x^{-3}$

$$= -{}^9C_7 \cdot \frac{1}{3^5 \cdot 2^2} \cdot x^{-3} = \frac{-1}{27}x^{-3}$$

$$(1 + 2x - 3x^3) \left( \frac{7}{18}x^0 - \frac{1}{27}x^{-3} \right)$$

$$\frac{7}{18} + \frac{3}{27} = \frac{7}{18} + \frac{1}{9} = \frac{7+2}{18} = \frac{9}{18} = \frac{1}{2}$$

$\therefore 108 \cdot \frac{1}{2} = 54$

23. Ans. (806)

Sol.  ${}^nC_1 x^{n-1} y = 135 \quad \dots(i)$

${}^nC_2 x^{n-2} y^2 = 30 \quad \dots(ii)$

${}^nC_3 x^{n-3} y^3 = \frac{10}{3} \quad \dots(iii)$

By  $\frac{(i)}{(ii)}$

$$\frac{{}^nC_1 x}{{}^nC_2 y} = \frac{9}{2} \quad \dots(iv)$$

By  $\frac{(ii)}{(iii)}$

$$\frac{{}^nC_2 x}{{}^nC_3 y} = 9 \quad \dots(v)$$

By  $\frac{(iv)}{(v)}$

$$\frac{{}^nC_1 C_3}{{}^nC_2 C_2} = \frac{1}{2}$$

$$\frac{2n^2(n-1)(n-2)}{6} = \frac{n(n-1)n(n-1)}{2 \cdot 2}$$

$$4n - 8 = 3n - 3$$

$$\Rightarrow \boxed{n = 5}$$

put in (v)

$$\frac{x}{y} = 9$$

$$x = 9y$$

put in (i)

$${}^5C_1 x^4 \left( \frac{x}{9} \right) = 135$$

$$x^5 = 27 \times 9$$

$$\Rightarrow x = 3, \quad y = \frac{1}{3}$$

$$6(n^3 + x^2 + y)$$

$$= 6 \left( 125 + 9 + \frac{1}{3} \right)$$

$$= 806$$

24. Ans. (5)

Sol.  $\alpha = \sum_{r=0}^n (4r^2 + 2r + 1) \cdot {}^nC_r$

$$\alpha = 4 \sum_{r=0}^n r^2 \cdot \frac{n}{r} \cdot {}^{n-1}C_{r-1} + 2 \sum_{r=0}^n r \cdot \frac{n}{r} \cdot {}^{n-1}C_{r-1} + \sum_{r=0}^n {}^nC_r$$

$$+ 4n \sum_{r=1}^n {}^{n-1}C_{r-1} + 2n \sum_{r=1}^n {}^{n-1}C_{r-1} + \sum_{r=0}^n {}^nC_r$$

$$\alpha = 4n(n-1) \cdot 2^{n-2} + 4n \cdot 2^{n-1} + 2n \cdot 2^{n-1} + 2^n$$

$$\alpha = 2^{n-2} [4n(n-1) + 8n + 4n + 4]$$

$$\alpha = 2^{n-2} [4n^2 + 8n + 4]$$

$$\alpha = 2n(n+1)^2$$

$$\beta = \sum_{r=0}^n \frac{{}^nC_r}{r+1} + \frac{1}{n+1} = \sum_{r=0}^n \frac{{}^{n+1}C_{r+1}}{n+1} + \frac{1}{n+1}$$

$$= \frac{1}{n+1} (1 + {}^{n+1}C_1 + \dots + {}^{n+1}C_{n+1})$$

$$= \frac{2^{n+1}}{n+1}$$

$$\frac{2\alpha}{\beta} = \frac{2^{n+1} (n+1)^2}{2^{n+1}} \cdot (n+1) = (n+1)^3$$

$$140 < (n+1)^3 < 281$$

$$n = 4 \Rightarrow (n+1)^3 = 125$$

$$n = 5 \Rightarrow (n+1)^3 = 216$$

$$n = 6 \Rightarrow (n+1)^3 = 343$$

$\therefore n = 5$