COMPLEX NUMBER

1. If $S = \{z \in C : |z - i| = |z + i| = |z - 1|\}$, then, n(S) is :

(1) 1 (2) 0

(3) 3 (4) 2

2. If α satisfies the equation $x^2 + x + 1 = 0$ and $(1 + \alpha)^7 = A + B\alpha + C\alpha^2$, A, B, C ≥ 0, then 5(3A – 2B – C) is equal to _____.

3. Let the complex numbers α and $\frac{1}{\overline{\alpha}}$ lie on the circles $|z - z_0|^2 = 4$ and $|z - z_0|^2 = 16$ respectively, where $z_0 = 1 + i$. Then, the value of $100|\alpha|^2$ is...........

4. If $z = \frac{1}{2} - 2i$, is such that

$|z + 1| = \alpha z + \beta(1 + i), i = \sqrt{-1}$ and $\alpha, \beta \in R$, then $\alpha + \beta$ is equal to

(1) –4 (2) 3

(3) 2 (4) –1

5. Let r and θ respectively be the modulus and amplitude of the complex number $z = 2 - i\left(2\tan\frac{5\pi}{8}\right)$, then (r, θ) is equal to

(1) $\left(2\sec\frac{3\pi}{8}, \frac{3\pi}{8}\right)$ (2) $\left(2\sec\frac{3\pi}{8}, \frac{5\pi}{8}\right)$

(3) $\left(2\sec\frac{5\pi}{8}, \frac{3\pi}{8}\right)$ (4) $\left(2\sec\frac{11\pi}{8}, \frac{11\pi}{8}\right)$

6. Let α, β be the roots of the equation $x^2 - \sqrt{6}x + 3 = 0$ such that Im (α) > Im (β). Let a, b be integers not divisible by 3 and n be a natural number such that

$\frac{\alpha^{99}}{\beta} + \alpha^{98} = 3^n(a + ib), i = \sqrt{-1}$.

Then n + a + b is equal to _______.

7. If z = x + iy, xy $\neq$ 0, satisfies the equation $z^2 + i\overline{z} = 0$, then $|z^2|$ is equal to :

(1) 9 (2) 1

(3) 4 (4) $\frac{1}{4}$

8. If z is a complex number, then the number of common roots of the equation $z^{1985} + z^{100} + 1 = 0$ and $z^3 + 2z^2 + 2z + 1 = 0$, is equal to :

(1) 1 (2) 2

(3) 0 (4) 3

9. If α denotes the number of solutions of $|1 - i|^x = 2^x$ and $\beta = \left(\frac{|z|}{\arg(z)}\right)$,

where $z = \frac{\pi}{4}(1 + i)^4\left(\frac{1 - \sqrt{\pi}i}{\sqrt{\pi} + i} + \frac{\sqrt{\pi} - i}{1 + \sqrt{\pi}i}\right)$,

$i = \sqrt{-1}$, then the distance of the point (α, β) from the line 4x – 3y = 7 is _____

10. Let z_1 and z_2 be two complex number such that $z_1 + z_2 = 5$ and $z_1^3 + z_2^3 = 20 + 15i$. Then $\left|z_1^4 + z_2^4\right|$ equals-

(1) $30\sqrt{3}$ (2) 75

(3) $15\sqrt{15}$ (4) $25\sqrt{3}$

11. Let $S = \{z \in C : |z - 1| = 1$ and $(\sqrt{2} - 1)(z + \overline{z}) - i(z - \overline{z}) = 2\sqrt{2}\}$. Let $z_1, z_2 \in S$ be such that $|z_1| = \max_{z \in s}|z|$ and $|z_2| = \min_{z \in s}|z|$.

Then $\left|\sqrt{2}z_1 - z_2\right|^2$ equals :

(1) 1 (2) 4

(3) 3 (4) 2

12. Let P = {z ∈ : $|z + 2 - 3i| \leq 1$} and Q = {z ∈ : $z(1 + i) + \overline{z}(1 - i) \leq -8$}. Let in P ∩ Q, $|z - 3 + 2i|$ be maximum and minimum at z_1 and z_2 respectively. If $|z_1|^2 + 2|z_2|^2 = \alpha + \beta\sqrt{2}$, where α, β are integers, then $\alpha + \beta$ equals ______.

13. If z is a complex number such that $|z| \geq 1$, then the minimum value of $\left|z+\frac{1}{2}(3+4i)\right|$ is:

(1) $\frac{5}{2}$ (2) 2

(3) 3 (4) $\frac{3}{2}$

14. The sum of all possible values of $\theta \in [-\pi, 2\pi]$, for which $\frac{1+i\cos\theta}{1-2i\cos\theta}$ is purely imaginary, is equal to

(1) 2π (2) 3π

(3) 5π (4) 4π

15. If the value of $\frac{3\cos 36^\circ + 5\sin 18^\circ}{5\cos 36^\circ - 3\sin 18^\circ}$ is $\frac{a\sqrt{5}-b}{c}$, where a, b, c are natural numbers and gcd(a, c) = 1, then a + b + c is equal to :

(1) 50 (2) 40

(3) 52 (4) 54

16. Let z be a complex number such that the real part of $\frac{z-2i}{z+2i}$ is zero. Then, the maximum value of $|z-(6+8i)|$ is equal to :

(1) 12 (2) ∞

(3) 10 (4) 8

17. The sum of the square of the modulus of the elements in the set $\{z = a+ib : a, b \in Z, z \in C, |z-1| \leq 1, |z-5| \leq |z-5i|\}$ is ________.

18. Let α and β be the sum and the product of all the non-zero solutions of the equation $(\bar{z})^2 + |z| = 0$, $z \in C$. Then $4(\alpha^2+\beta^2)$ is equal to :

(1) 6 (2) 4

(3) 8 (4) 2

19. The area (in sq. units) of the region $S = \{z \in \;; |z-1| \leq 2; (z+\bar{z}) + i(z-\bar{z}) \leq 2, \operatorname{Im}(z) \geq 0\}$ is

(1) $\frac{7\pi}{3}$ (2) $\frac{3\pi}{2}$

(3) $\frac{17\pi}{8}$ (4) $\frac{7\pi}{4}$

20. Let $S_1 = \{z \in C : |z| \leq 5\}$,

$S_2 = \left\{z \in C : \operatorname{Im}\left(\frac{z+1-\sqrt{3}\,i}{1-\sqrt{3}\,i}\right) \geq 0\right\}$ and

$S_3 = \{z \in C : \operatorname{Re}(z) \geq 0\}$.

Then the area of region $S_1 \cap S_2 \cap S_3$ is

(1) $\frac{125\pi}{6}$ (2) $\frac{125\pi}{24}$

(3) $\frac{125\pi}{4}$ (4) $\frac{125\pi}{12}$

21. If z_1, z_2 are two distinct complex number such that $\left|\frac{z_1-2z_2}{\frac{1}{2}-z_1\bar{z}_2}\right| = 2$, then

(1) either z_1 lies on a circle of radius 1 or z_2 lies on a circle of radius $\frac{1}{2}$

(2) either z_1 lies on a circle of radius $\frac{1}{2}$ or z_2 lies on a circle of radius 1.

(3) z_1 lies on a circle of radius $\frac{1}{2}$ and z_2 lies on a circle of radius 1.

(4) both z_1 and z_2 lie on the same circle.

22. Consider the following two statements :

Statement I : For any two non-zero complex numbers z_1, z_2

$(|z_1|+|z_2|)\left|\frac{z_1}{|z_1|}+\frac{z_2}{|z_2|}\right| \leq 2(|z_1|+|z_2|)$ and

Statement II : If x, y, z are three distinct complex numbers and a, b, c are three positive real numbers such that $\frac{a}{|y-z|} = \frac{b}{|z-x|} = \frac{c}{|x-y|}$, then

$\frac{a^2}{y-z}+\frac{b^2}{z-x}+\frac{c^2}{x-y} = 1$.

Between the above two statements,

(1) both Statement I and Statement II are incorrect.

(2) Statement I is incorrect but Statement II is correct.

(3) Statement I is correct but Statement II is incorrect.

(4) both Statement I and Statement II are correct.

23. Let z be a complex number such that $|z + 2| = 1$ and $\operatorname{Im}\left(\frac{z+1}{z+2}\right) = \frac{1}{5}$. Then the value of $\left|\operatorname{Re}\left(\overline{z+2}\right)\right|$ is :

(1) $\frac{\sqrt{6}}{5}$ (2) $\frac{1+\sqrt{6}}{5}$

(3) $\frac{24}{5}$ (4) $\frac{2\sqrt{6}}{5}$

24. If the set R = {(a, b) ; a + 5b = 42, a, b $\in$ }

has m elements and $\sum_{n=1}^{m}\left(1+i^{n!}\right)$ = x + iy, where

$i = \sqrt{-1}$, then the value of m + x + y is :

(1) 8 (2) 12

(3) 4 (4) 5

25. Let x_1, x_2, x_3, x_4 be the solution of the equation $4x^4 + 8x^3 - 17x^2 - 12x + 9 = 0$ and

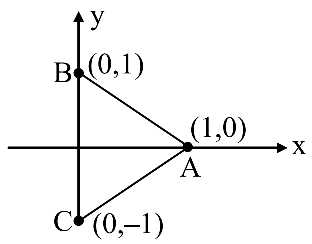
$\left(4+x_1^2\right)\left(4+x_2^2\right)\left(4+x_3^2\right)\left(4+x_4^2\right) = \frac{125}{16}m$.

Then the value of m is ________.

SOLUTIONS

1. Ans. (1)

Sol. $|z - i| = |z + i| = |z - 1|$



ABC is a triangle. Hence its circum-centre will be the only point whose distance from A, B, C will be same.

So n(S) = 1

2. Ans. (5)

Sol. $x^2 + x + 1 = 0 \Rightarrow x = \omega, \omega^2 = \alpha$

Let $\alpha = \omega$

Now $(1 + \alpha)^7 = -\omega^{14} = -\omega^2 = 1 + \omega$

A = 1, B = 1, C = 0

$\therefore 5(3A - 2B - C) = 5(3 - 2 - 0) = 5$

3. Ans. (20)

Sol. $|z - z_0|^2 = 4$

$\Rightarrow (\alpha - z_0)(\bar{\alpha} - \bar{z}_0) = 4$

$\Rightarrow \alpha\bar{\alpha} - \alpha\bar{z}_0 - z_0\bar{\alpha} + |z_0|^2 = 4$

$\Rightarrow |\alpha|^2 - \alpha\bar{z}_0 - z_0\bar{\alpha} = 2$ ……(1)

$|z - z_0|^2 = 16$

$\Rightarrow \left(\frac{1}{\bar{\alpha}} - z_0\right)\left(\frac{1}{\alpha} - \bar{z}_0\right) = 16$

$\Rightarrow (1 - \bar{\alpha}z_0)(1 - \alpha\bar{z}_0) = 16|\alpha|^2$

$\Rightarrow 1 - \bar{\alpha}z_0 - \alpha\bar{z}_0 + |\alpha|^2|z_0|^2 = 16|\alpha|^2$

$\Rightarrow 1 - \bar{\alpha}z_0 - \alpha\bar{z}_0 = 14|\alpha|^2$ ……(2)

From (1) and (2)

$\Rightarrow 5|\alpha|^2 = 1$

$\Rightarrow 100|\alpha|^2 = 20$

4. Ans. (2)

Sol. $z = \frac{1}{2} - 2i$

$|z + 1| = \alpha z + \beta(1 + i)$

$\left|\frac{3}{2} - 2i\right| = \frac{\alpha}{2} - 2\alpha i + \beta + \beta i$

$\left|\frac{3}{2} - 2i\right| = \left(\frac{\alpha}{2} + \beta\right) + (\beta - 2\alpha)i$

$\beta = 2\alpha$ and $\frac{\alpha}{2} + \beta = \sqrt{\frac{9}{4} + 4}$

$\alpha + \beta = 3$

5. Ans. (1)

Sol. $z = 2 - i\left(2\tan\frac{5\pi}{8}\right) = x + iy$ (let)

$r = \sqrt{x^2 + y^2}$ & $\theta = \tan^{-1}\frac{y}{x}$

$r = \sqrt{(2)^2 + \left(2\tan\frac{5\pi}{8}\right)^2}$

$= \left|2\sec\frac{5\pi}{8}\right| = \left|2\sec\left(\pi - \frac{3\pi}{8}\right)\right|$

$= 2\sec\frac{3\pi}{8}$

& $\theta = \tan^{-1}\left(\frac{-2\tan\frac{5\pi}{8}}{2}\right)$

$= \tan^{-1}\left(\tan\left(\pi - \frac{5\pi}{8}\right)\right)$

$= \frac{3\pi}{8}$

6. **Ans. (49)**

Sol. $x^2 - \sqrt{6}x + 6 = 0$ $\begin{matrix}\alpha \\ \beta\end{matrix}$

$x = \frac{\sqrt{6} \pm i\sqrt{6}}{2} = \frac{\sqrt{6}}{2}(1 \pm i)$

$\alpha = \sqrt{3}(e^{i\frac{\pi}{4}})$, $\beta = \sqrt{3}(e^{-i\frac{\pi}{4}})$

$\therefore \frac{\alpha^{99}}{\beta} + \alpha^{98} = \alpha^{98}\left(\frac{\alpha}{\beta} + 1\right)$

$= \frac{\alpha^{98}(\alpha + \beta)}{\beta} = 3^{49}\left(e^{i99\frac{\pi}{4}}\right).\ \sqrt{2}$

$= 3^{49}(-1 + i)$

$= 3^n(a + ib)$

$\therefore$ n = 49, a = –1, b = 1

$\therefore$ n + a + b = 49 – 1 + 1 = 49

7. **Ans. (2)**

Sol. $z^2 = -i\bar{z}$

$\left|z^2\right| = \left|i\bar{z}\right|$

$|z^2| = |z|$

$|z|^2 - |z| = 0$

$|z|(|z| - 1) = 0$

$|z| = 0$ (not acceptable)

$\therefore |z| = 1$

$\therefore |z|^2 = 1$

8. **Ans. (2)**

Sol. $z^{1985} + z^{100} + 1 = 0$ & $z^3 + 2z^2 + 2z + 1 = 0$

$(z + 1)(z^2 - z + 1) + 2z(z + 1) = 0$

$(z + 1)(z^2 + z + 1) = 0$

$\Rightarrow z = -1,\ z = w, w^2$

Now putting z = –1 not satisfy

Now put z = w

$\Rightarrow w^{1985} + w^{100} + 1$

$\Rightarrow w^2 + w + 1 = 0$

Also, $z = w^2$

$\Rightarrow w^{3970} + w^{200} + 1$

$\Rightarrow w + w^2 + 1 = 0$

Two common root

9. **Ans. (3)**

Sol. $\left(\sqrt{2}\right)^x = 2^x \Rightarrow x = 0 \Rightarrow \alpha = 1$

$z = \frac{\pi}{4}(1 + i)^4\left[\frac{\sqrt{\pi} - \pi i - i - \sqrt{\pi}}{\pi + 1} + \frac{\sqrt{\pi} - i - \pi i - \sqrt{\pi}}{1 + \pi}\right]$

$= -\frac{\pi i}{2}\left(1 + 4i + 6i^2 + 4i^3 + 1\right)$

$= 2\pi i$

$\beta = \frac{2\pi}{\frac{\pi}{2}} = 4$

Distance from (1, 4) to $4x - 3y = 7$

Will be $\frac{15}{5} = 3$

10. **Ans. (2)**

Sol. $z_1 + z_2 = 5$

$z_1^3 + z_2^3 = 20 + 15i$

$z_1^3 + z_2^3 = (z_1 + z_2)^3 - 3z_1z_2(z_1 + z_2)$

$z_1^3 + z_2^3 = 125 - 3z_1 . z_2(5)$

$\Rightarrow 20 + 15i = 125 - 15z_1z_2$

$\Rightarrow 3z_1z_2 = 25 - 4 - 3i$

$\Rightarrow 3z_1z_2 = 21 - 3i$

$\Rightarrow z_1.z_2 = 7 - i$

$\Rightarrow (z_1 + z_2)^2 = 25$

$\Rightarrow z_1^2 + z_2^2 = 25 - 2(7 - i)$

$\Rightarrow 11 + 2i$

$(z_1^2 + z_2^2)^2 = 121 - 4 + 44i$

$\Rightarrow z_1^4 + z_2^4 + 2(7 - i)^2 = 117 + 44i$

$\Rightarrow z_1^4 + z_2^4 = 117 + 44i - 2(49 - 1 - 14i)$

$\Rightarrow \left|z_1^4 + z_2^4\right| = 75$

11. Ans. (4)

Sol. Let Z = x + iy

Then $(x-1)^2 + y^2 = 1$ ……(1)

& $\left(\sqrt{2}-1\right)\left(2x\right) - i(2iy) = 2\sqrt{2}$

$\Rightarrow (\sqrt{2}-1)x + y = \sqrt{2}$ ……(2)

Solving (1) & (2) we get

Either x = 1 or $x = \frac{1}{2-\sqrt{2}}$ ……(3)

On solving (3) with (2) we get

For x = 1 $\Rightarrow$ y = 1 $\Rightarrow$ $Z_2 = 1 + i$

& for

$x = \frac{1}{2-\sqrt{2}} \Rightarrow y \quad \sqrt{2} - \frac{1}{\sqrt{2}}$

$\Rightarrow Z_1 = \left(1+\frac{1}{\sqrt{2}}\right) + \frac{i}{\sqrt{2}}$

Now

$\left|\sqrt{2}z_1 - z_2\right|^2$

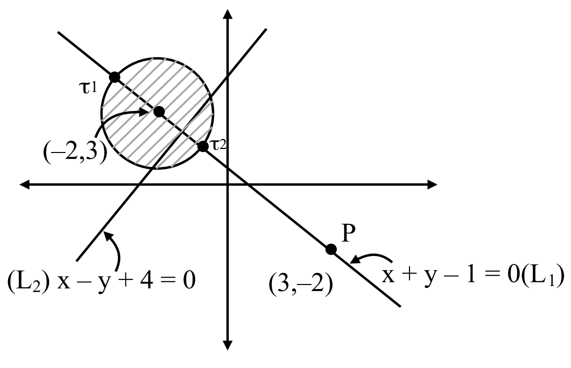
$= \left|\left(\frac{1}{\sqrt{2}}+1\right)\sqrt{2} + i - (1+i)\right|^2$

$= \left(\sqrt{2}\right)^2$

= 2

12. Ans. (36)

Sol.



Clearly for the shaded region z_1 is the intersection of the circle and the line passing through P (L and z_2 is intersection of line L_1 & L_2

Circle : $(x+2)^2 + (y-3)^2 = 1$

$L_1 : x + y - 1 = 0$

$L_2 : x - y + 4 = 0$

On solving circle & L_1 we get

$z_1 : \left(-2-\frac{1}{\sqrt{2}}, 3+\frac{1}{\sqrt{2}}\right)$

On solving L_1 and z_2 is intersection of line L_1 & L_2 we get $z_2 : \left(\frac{-3}{2}, \frac{5}{2}\right)$

$|z_1|^2 + 2|z_2|^2 = 14 + 5\sqrt{2} + 17$

$= 31 + 5\sqrt{2}$

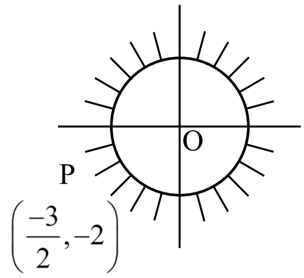
So $\alpha = 31$

$\beta = 5$

$\alpha + \beta = 36$

13. Ans. (Bonus)

Sol. $|z| \geq 1$



Min. value of $\left|z + \frac{3}{2} + 2i\right|$ is actually zero.

14. Ans. (2)

Sol. $Z = \frac{1 + i\cos\theta}{1 - 2i\cos\theta}$

$Z = -\bar{Z} \Rightarrow \frac{1+i\cos\theta}{1-2i\cos\theta} = -\left(\overline{\frac{1+i\cos\theta}{1-2i\cos\theta}}\right)$

$(1+i\cos\theta)\left(\overline{1-2i\cos\theta}\right) = -(1-2i\cos\theta)\left(\overline{1+i\cos\theta}\right)$

$(1+i\cos\theta)(1+2i\cos\theta) = -(1-2i\cos\theta)(1-i\cos\theta)$

$1 + 3i\cos\theta - 2\cos^2\theta = -(1 - 3i\cos\theta - 2\cos^2\theta)$

$2 - 4\cos^2\theta = 0$

$\Rightarrow \cos^2\theta = \frac{1}{2} \Rightarrow \theta = -\frac{\pi}{4}, -\frac{3\pi}{4}, \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

sum = 3π

15. Ans. (3)

Sol. $\dfrac{\dfrac{3(\sqrt{5}+1)}{4}+5\left(\dfrac{\sqrt{5}-1}{4}\right)}{5\left(\dfrac{\sqrt{5}+1}{4}\right)-3\left(\dfrac{\sqrt{5}-1}{4}\right)}=\dfrac{8\sqrt{5}-2}{2\sqrt{5}+8}$

$=\dfrac{4\sqrt{5}-1}{\sqrt{5}+4}\cdot\dfrac{\sqrt{5}-4}{\sqrt{5}-4}$

$=\dfrac{20-16\sqrt{5}-\sqrt{5}+4}{-11}$

$=\dfrac{17\sqrt{5}-24}{11}\Rightarrow$ a = 17, b = 27, c = 11

a + b + c = 52

16. Ans. (1)

Sol. $\dfrac{z-2i}{z+2i}+\dfrac{\bar{z}+2i}{\bar{z}-2i}=0$

$z\bar{z}-2i\bar{z}-2iz+4(-1)$

$+z\bar{z}+2zi+2\bar{z}i+4(-1)=0$

$\Rightarrow 2|z|^2 = 8 \Rightarrow |z| = 2$

$|z-(6+8i)|_{\text{maximum}}=10+2=12$

17. Ans. (9)

Sol. $|z-1|\le 1$

$\Rightarrow |(x-1)+iy|\le 1$

$\Rightarrow \sqrt{(x-1)^2+y^2}\le 1$

$\Rightarrow (x-1)^2+y^2\le 1$ .... (1)

Also $|z-5|\le|z-5i|$

$(x-5)^2+y^2\le x^2+(y-5)^2$

$-10x\le -10y$

$\Rightarrow x\ge y$ .... (2)

Solving (1) and (2)

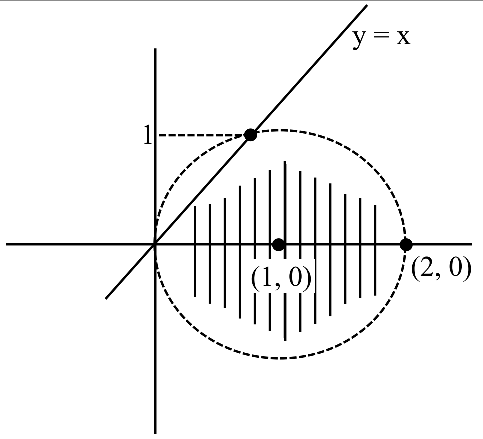
$\Rightarrow (x-1)^2+x^2=1$

$\Rightarrow 2x^2-2x=0$

$\Rightarrow x(x-1)=0$

$\Rightarrow x=0$ or $x=1$

$y=0$ or $y=1$



Given x, y ∈ I

Points (0, 0), (1, 0), (2, 0), (1, 1), (1, –1)

to find

$|z_1|^2+|z_2|^2+|z_3|^2+|z_4|^2+|z_5|^2$

= 0 + 1 + 4 + 1 + 1 + 1 + 1 = 9

18. Ans. (2)

Sol. z = x + iy

$\bar{z}=x-iy$

$\bar{z}^2=x^2-y^2-2ixy$

$\Rightarrow x^2-y^2-2ixy+\sqrt{x^2+y^2}=0$

$\Rightarrow x = 0$ or $y = 0$

$-y^2 + |y| = 0$ $x^2 + |x| = 0$

$|y| = |y|^2$ $\Rightarrow x = 0$

y = 0, ±1

$\Rightarrow$ i, –i $\Rightarrow \alpha = i - i = 0$

are roots $\beta = i(-i) = 1$

19. Ans. (2)

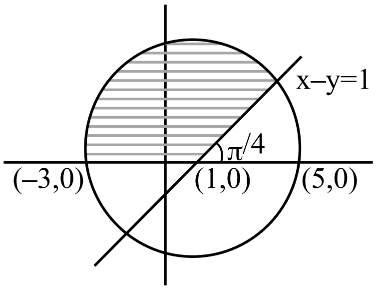
Sol. Put z = x + iy

$|z-1|\le 2\Rightarrow (x-1)^2+y^2\le 4$ …(1)

$(z+\bar{z})+i(z-\bar{z})\le 2\Rightarrow 2x+i(2iy)\le 2$

$\Rightarrow x - y\le 1$ …(2)

$\text{Im}(z)\ge 0\Rightarrow y\ge 0$ …(3)



Required area

= Area of semi-circle – area of sector A

$\frac{1}{2}\pi(2)^2 - \frac{\pi}{2}$

$= \frac{3\pi}{2}$

20. Ans. (4)

Sol. $S_1 : x^2 + y^2 \le 25$ .....(1)

$S_2 : \text{lm of } \frac{z + \left(1 - \sqrt{3}\,i\right)}{\left(1 - \sqrt{3}\,i\right)} \ge 0$

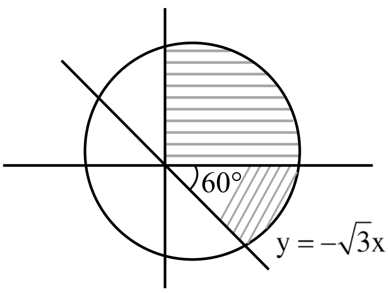
$\text{lm of } \left(\frac{x + iy}{1 - \sqrt{3}\,i} + 1\right) \ge 0$

$\text{lm of } \left(\frac{(x + iy)\left(1 + \sqrt{3}\,i\right)}{4}\right) \ge 0$

$\Rightarrow \sqrt{3}\,x + y \ge 0$ ......(2)

$S_3 : x \ge 0$ ......(3)

$\text{Area} = \frac{5}{12}\left(\pi(5)^2\right)$



21. Ans. (1)

Sol. $\frac{z_1 - 2z_2}{\frac{1}{2} - z_1\bar{z}_2} \cdot \frac{\bar{z}_1 - 2\bar{z}_2}{\frac{1}{2} - \bar{z}_1 z_2} = 4$

$|z_1|^2\ 2z_1\bar{z}_2 - 2\bar{z}_1 z_2 + 4|z_2|^2$

$= 4\left(\frac{1}{4} - \frac{\bar{z}_1 z_2}{2} - \frac{z_1\bar{z}_2}{2} + |z_1|^2|z_2|^2\right)$

$z_1\bar{z}_1 + 2z_2\ .2\bar{z}_2 - z_1\bar{z}_1 2z_2\ 2\bar{z}_2 - 1 = 0$

$\left(z, \bar{z}_1 - 1\right)\left(1 - 2z_2\ .2\bar{z}_2\right) = 0$

$\left(|z_1|^2 - 1\right)\left(|2z_2|^2 - 1\right) = 0$

22. Ans. (3)

Sol. Statement I :

$\left(|z_1| + |z_2|\right)\left|\frac{z_1}{|z_1|} + \frac{z_2}{|z_2|}\right|$

Since $\left|\frac{z_1}{|z_1|} + \frac{z_2}{|z_2|}\right| \le \left|\frac{z_1}{|z_1|}\right| + \left|\frac{z_2}{|z_2|}\right|$

$\left|\frac{z_1}{|z_1|} + \frac{z_2}{|z_2|}\right| \le \frac{|z_1|}{|z_1|} + \frac{|z_2|}{|z_2|}$

$\left|\frac{z_1}{|z_1|} + \frac{z_2}{|z_2|}\right| \le 2$

$\left(|z_1| + |z_2|\right)\left(\left|\frac{z_1}{|z_1|} + \frac{z_2}{|z_2|}\right|\right) \le 2\left(|z_1| + |z_2|\right)$

$\therefore$ statement I is correct

For Statement II :

$\frac{a}{|y - z|} = \frac{b}{|z - x|} = \frac{c}{|x - y|}$

$\frac{a^2}{|y - z|^2} = \frac{b^2}{|z - x|^2} = \frac{c^2}{|x - y|^2} = \lambda$

$a^2 = \lambda(|y - z|^2) = \lambda(y - z)\left(\bar{y} - \bar{z}\right)$

$b^2 = \lambda(z - x)\left(\bar{z} - \bar{x}\right)$ and $c^2 = \lambda(x - y)\left(\bar{x} - \bar{y}\right)$

$\frac{a^2}{y - z} + \frac{b^2}{z - x} + \frac{c^2}{x - y} = \lambda\left(\bar{y} - \bar{z} + \bar{z} - \bar{x} + \bar{x} - \bar{y}\right) = 0$

Statement II is false

23. Ans. (4)

Sol. $|z + 2| = 1$, $\text{Im}\left(\frac{z+1}{z+2}\right) = \frac{1}{5}$

Let $z + 2 = \cos\theta + i\sin\theta$

$\frac{1}{z+2} = \cos\theta - i\sin\theta$

$\Rightarrow \frac{z+1}{z+2} = 1 - \frac{1}{z+2} = 1 - (\cos\theta - i\sin\theta)$

$= (1 - \cos\theta) + i\sin\theta$

$\text{Im}\left(\frac{z+1}{z+2}\right) = \sin\theta$, $\sin\theta = \frac{1}{5}$

$\cos\theta = \pm\sqrt{1 - \frac{1}{25}} = \pm\frac{2\sqrt{6}}{5}$

$\left|\text{Re}(\overline{z+2})\right| = \frac{2\sqrt{6}}{5}$

24. Ans. (2)

Sol. $a + 5b = 42$, $a, b \in N$

$a = 42 - 5b$, $b = 1$, $a = 37$

$b = 2$, $a = 32$

$b = 3$, $a = 27$

$b = 8$, $a = 2$

R has "8" elements $\Rightarrow$ $m = 8$

$\sum_{n=1}^{8}(1 - i^{n!}) = x + iy$

for $n \geq 4$, $i^{n!} = 1$

$\Rightarrow (1 - i) + (1 - i^{2!}) + (1 - i^{3!})$

$= 1 - I + 2 + 1 + 1$

$= 5 - I = x + iy$

$m + x + y = 8 + 5 - 1 = 12$

25. Ans. (221)

Sol. $4x^4 + 8x^3 - 17x^2 - 12x + 9$

$= 4(x - x_1)(x - x_2)(x - x_3)(x - x_4)$

Put $x = 2i$ & $-2i$

$64 - 64i + 68 - 24i + 9 = (2i - x_1)(2i - x_2)(2i - x_3)(2i - x_4)$

$= 141 - 88i$ ......(1)

$64 + 64i + 68 + 24i + 9 = 4(-2i - x_1)(-2i - x_2)$

$(-2i - x_3)(-2i - x_4)$

$= 141 + 88i$ ....(2)

$\frac{125}{16}m = \frac{141^2 + 88^2}{16}$

$m = 221$