DIFFERENTIAL EQUATION

1. Let x = x(t) and y = y(t) be solutions of the differential equations $\frac{dx}{dt}+ax=0$ and $\frac{dy}{dt}+by=0$ respectively, a, b $\in$ R. Given that x(0) = 2; y(0) = 1 and 3y(1) = 2x(1), the value of t, for which x(t) = y(t), is :

(1) $\log_{\frac{2}{3}}2$ (2) $\log_4 3$

(3) $\log_3 4$ (4) $\log_{\frac{4}{3}}2$

2. If the solution of the differential equation (2x + 3y – 2) dx + (4x + 6y – 7) dy = 0, y(0) = 3, is $\alpha x+\beta y+3\log_e|2x+3y-\gamma|=6$, then $\alpha+2\beta+3\gamma$ is equal to _____.

3. If y = y(x) is the solution curve of the differential equation $(x^2-4)\,dy-(y^2-3y)dx=0$, $x>2,\ y(4)=\frac{3}{2}$ and the slope of the curve is never zero, then the value of y(10) equals :

(1) $\frac{3}{1+(8)^{1/4}}$ (2) $\frac{3}{1+2\sqrt{2}}$

(3) $\frac{3}{1-2\sqrt{2}}$ (4) $\frac{3}{1-(8)^{1/4}}$

4. If the solution curve, of the differential equation $\frac{dy}{dx}=\frac{x+y-2}{x-y}$ passing through the point (2, 1) is

$$\tan^{-1}\left(\frac{y-1}{x-1}\right)-\frac{1}{\beta}\log_e\left(\alpha+\left(\frac{y-1}{x-1}\right)^2\right)=\log_e|x-1|,$$

then $5\beta+\alpha$ is equal to

5. A function y = f(x) satisfies $f(x)\sin 2x+\sin x-(1+\cos^2 x)f'(x)=0$ with condition f(0) = 0 . Then $f\left(\frac{\pi}{2}\right)$ is equal to

(1) 1 (2) 0 (3) –1 (4) 2

6. If the solution curve $y=y(x)$ of the differential equation $(1+y^2)(1+\log_e x)dx+x\,dy=0,\ x>0$ passes through the point (1, 1) and $y(e)=\frac{\alpha-\tan\left(\frac{3}{2}\right)}{\beta+\tan\left(\frac{3}{2}\right)}$, then $\alpha+2\beta$ is

7. If $\sin\left(\frac{y}{x}\right)=\log_e|x|+\frac{\alpha}{2}$ is the solution of the differential equation $x\cos\left(\frac{y}{x}\right)\frac{dy}{dx}=y\cos\left(\frac{y}{x}\right)+x$ and $y(1)=\frac{\pi}{3}$, then α^2 is equal to

(1) 3 (2) 12 (3) 4 (4) 9

8. Let

$$f(x)=\sqrt{\lim_{r\to x}\left\{\frac{2r^2\left[(f(r))^2-f(x)f(r)\right]}{r^2-x^2}-r^3e^{\frac{f(r)}{r}}\right\}}$$

be differentiable in $(-\infty,0)\cup(0,\infty)$ and f(1) = 1. Then the value of ea, such that f(a) = 0, is equal to _____.

9. Let y = y (x) be the solution of the differential equation sec x dy + {2(1 – x) tan x + x(2 – x)} dx = 0 such that y(0) = 2.Then y(2) is equal to :

(1) 2 (2) 2{1 – sin (2)}

(3) 2{sin (2) + 1} (4) 1

10. Let y = y(x) be the solution of the differential equation $(1-x^2)\,dy=\left[xy+(x^3+2)\sqrt{3(1-x^2)}\right]dx$, $-1<x<1,\ y(0)=0$. If $y\left(\frac{1}{2}\right)=\frac{m}{n}$, m and n are co-prime numbers, then m + n is equal to _______.

11. Let Y = Y(X) be a curve lying in the first quadrant such that the area enclosed by the line Y – y = Y ′(x) (X – x) and the co-ordinate axes, where (x, y) is any point on the curve, is always $\frac{-y^2}{2Y'(x)}+1$, $Y'(x)\neq 0$. If Y(1) = 1, then 12Y(2) equals ______ .

12. The solution curve of the differential equation

$$y\frac{dx}{dy} = x\left(\log_e x - \log_e y + 1\right),\ x > 0, y > 0$$

passing through the point (e, 1) is

(1) $\left|\log_e \frac{y}{x}\right| = x$ (2) $\left|\log_e \frac{y}{x}\right| = y^2$

(3) $\left|\log_e \frac{x}{y}\right| = y$ (4) $2\left|\log_e \frac{x}{y}\right| = y + 1$

13. Let y = y(x) be the solution of the differential equation $\frac{dy}{dx} = \frac{(\tan x) + y}{\sin x(\sec x - \sin x \tan x)}$, $x \in \left(0, \frac{\pi}{2}\right)$ satisfying the condition $y\left(\frac{\pi}{4}\right) = 2$. Then, $y\left(\frac{\pi}{3}\right)$ is

(1) $\sqrt{3}\left(2 + \log_e \sqrt{3}\right)$ (2) $\frac{\sqrt{3}}{2}\left(2 + \log_e 3\right)$

(3) $\sqrt{3}\left(1 + 2\log_e 3\right)$ (4) $\sqrt{3}\left(2 + \log_e 3\right)$

14. The temperature T(t) of a body at time t = 0 is 160^o F and it decreases continuously as per the differential equation $\frac{dT}{dt} = -K(T - 80)$, where K is positive constant. If T(15) = 120^oF, then T(45) is equal to

(1) 85^o F (2) 95^o F

(3) 90^o F (4) 80^o F

15. Let y = y(x) be the solution of the differential equation $\sec^2 x dx + \left(e^{2y}\tan^2 x + \tan x\right)dy = 0$, $0 < x < \frac{\pi}{2}$, $y\left(\frac{\pi}{4}\right) = 0$. If $y\left(\frac{\pi}{6}\right) = \alpha$.

Then $e^{8\alpha}$ is equal to ______.

16. Let y = y(x) be the solution of the differential equation $\frac{dy}{dx} = 2x(x + y)^3 - x(x + y) - 1$, y(0) = 1.

Then, $\left(\frac{1}{\sqrt{2}} + y\left(\frac{1}{\sqrt{2}}\right)\right)^2$ equals :

(1) $\frac{4}{4 + \sqrt{e}}$ (2) $\frac{3}{3 - \sqrt{e}}$

(3) $\frac{2}{1 + \sqrt{e}}$ (4) $\frac{1}{2 - \sqrt{e}}$

17. If x = x(t) is the solution of the differential equation $(t + 1)dx = (2x + (t + 1)^4)\,dt$, x(0) = 2, then, x(1) equals __________.

18. Let α be a non-zero real number. Suppose $f: \mathbb{R} \to \mathbb{R}$ is a differentiable function such that $f(0) = 2$ and $\lim_{x \to -\infty} f(x) = 1$. If $f'(x) = \alpha f(x) + 3$, for all $x \in \mathbb{R}$, then $f(-\log_e 2)$ is equal to____.

(1) 3 (2) 5

(3) 9 (4) 7

19. If $\frac{dx}{dy} = \frac{1 + x - y^2}{y}$, x(1) = 1, then 5x(2) is equal to :

20. Let y = y(x) be the solution curve of the differential equation $\sec y\frac{dy}{dx} + 2x\sin y = x^3\cos y$, y(1) = 0. Then $y\left(\sqrt{3}\right)$ is equal to :

(1) $\frac{\pi}{3}$ (2) $\frac{\pi}{6}$

(3) $\frac{\pi}{4}$ (4) $\frac{\pi}{12}$

21. Let $\alpha|x| = |y|e^{xy - \beta}$, $\alpha, \beta \in \mathbb{N}$ be the solution of the differential equation $xdy - ydx + xy(xdy + ydx) = 0$, y(1) = 2. Then $\alpha + \beta$ is equal to _____

22. Let $\int_0^x \sqrt{1 - (y'(t))^2}\,dt = \int_0^x y(t)dt$, $0 \le x \le 3$, $y \ge 0$, y (0) = 0. Then at x = 2, y'' + y + 1 is equal to :

(1) 1 (2) 2

(3) $\sqrt{2}$ (4) 1/2

23. If $\log_e y = 3\sin^{-1}x$, then $(1 - x^2)y'' - xy'$ at $x = \frac{1}{2}$ is equal to :

(1) $9e^{\pi/6}$ (2) $3e^{\pi/6}$

(3) $3e^{\pi/2}$ (4) $9e^{\pi/2}$

24. For a differentiable function $f: \mathbb{R} \to \mathbb{R}$, suppose $f'(x) = 3f(x) + \alpha$, where $\alpha \in \mathbb{R}$, $f(0) = 1$ and $\lim_{x \to -\infty} f(x) = 7$. Then $9f(-\log_e 3)$ is equal to______.

25. The solution curve, of the differential equation $2y\frac{dy}{dx} + 3 = 5\frac{dy}{dx}$, passing through the point (0, 1) is a conic, whose vertex lies on the line :

(1) $2x + 3y = 9$ (2) $2x + 3y = -9$

(3) $2x + 3y = -6$ (4) $2x + 3y = 6$

26. The solution of the differential equation $(x^2 + y^2)dx - 5xy\,dy = 0$, $y(1) = 0$, is :

(1) $|x^2 - 4y^2|^5 = x^2$ (2) $|x^2 - 2y^2|^6 = x$

(3) $|x^2 - 4y^2|^6 = x$ (4) $|x^2 - 2y^2|^5 = x^2$

27. If the solution $y = y(x)$ of the differential equation $(x^4 + 2x^3 + 3x^2 + 2x + 2)dy - (2x^2 + 2x + 3)dx = 0$ satisfies $y(-1) = -\frac{\pi}{4}$, then $y(0)$ is equal to :

(1) $-\frac{\pi}{12}$ (2) 0

(3) $\frac{\pi}{4}$ (4) $\frac{\pi}{2}$

28. Let the solution $y = y(x)$ of the differential equation $\frac{dy}{dx} - y = 1 + 4\sin x$ satisfy $y(\pi) = 1$. Then $y\left(\frac{\pi}{2}\right) + 10$ is equal to ______

29. The differential equation of the family of circles passing the origin and having center at the line $y = x$ is :

(1) $(x^2 - y^2 + 2xy)dx = (x^2 - y^2 + 2xy)dy$

(2) $(x^2 + y^2 + 2xy)dx = (x^2 + y^2 - 2xy)dy$

(3) $(x^2 - y^2 + 2xy)dx = (x^2 - y^2 - 2xy)dy$

(4) $(x^2 + y^2 - 2xy)dx = (x^2 + y^2 + 2xy)dy$

30. Let $y = y(x)$ be the solution of the differential equation $\frac{dy}{dx} + \frac{2x}{(1+x^2)^2}y = xe^{\frac{1}{(1+x^2)}}$; $y(0) = 0$. Then the area enclosed by the curve $f(x) = y(x)e^{-\frac{1}{(1+x^2)}}$ and the line $y - x = 4$ is ____.

31. Let $y = y(x)$ be the solution of the differential equation $(x^2 + 4)^2dy + (2x^3y + 8xy - 2)dx = 0$. If $y(0) = 0$, then $y(2)$ is equal to

(1) $\frac{\pi}{8}$ (2) $\frac{\pi}{16}$

(3) 2π (4) $\frac{\pi}{32}$

32. Let $y = y(x)$ be the solution of the differential equation $(x + y + 2)^2\,dx = dy$, $y(0) = -2$. Let the maximum and minimum values of the function $y = y(x)$ in $\left[0, \frac{\pi}{3}\right]$ be α and β, respectively. If $(3\alpha + \pi)^2 + \beta^2 = \gamma + \delta\sqrt{3}$, $\gamma, \delta \in$, then $\gamma + \delta$ equals …..

33. If $y = y(x)$ is the solution of the differential equation $\frac{dy}{dx} + 2y = \sin(2x)$, $y(0) = \frac{3}{4}$, then $y\left(\frac{\pi}{8}\right)$ is equal to :

(1) $e^{-\pi/8}$ (2) $e^{-\pi/4}$

(3) $e^{\pi/4}$ (4) $e^{\pi/8}$

34. Suppose the solution of the differential equation $\frac{dy}{dx} = \frac{(2+\alpha)x - \beta y + 2}{\beta x - 2\alpha y - (\beta\gamma - 4\alpha)}$ represents a circle passing through origin. Then the radius of this circle is :

(1) $\sqrt{17}$ (2) $\frac{1}{2}$

(3) $\frac{\sqrt{17}}{2}$ (4) 2

35. If the solution $y(x)$ of the given differential equation $(e^y + 1)\cos x\,dx + e^y \sin x\,dy = 0$ passes through the point $\left(\frac{\pi}{2}, 0\right)$, then the value of $e^{y\left(\frac{\pi}{6}\right)}$ is equal to ______.

36. Let f be a differentiable function in the interval $(0, \infty)$ such that $f(1) = 1$ and $\lim_{t \to x} \frac{t^2 f(x) - x^2 f(t)}{t - x} = 1$ for each $x > 0$. Then $2f(2) + 3f(3)$ is equal to _______.

37. Let f(x) be a positive function such that the area bounded by y = f(x), y = 0 from x = 0 to x = a > 0 is $e^{-a} + 4a^2 + a - 1$. Then the differential equation, whose general solution is $y = c_1 f(x) + c_2$, where c_1 and c_2 are arbitrary constants, is :

(1) $(8e^x - 1)\frac{d^2y}{dx^2} + \frac{dy}{dx} = 0$

(2) $(8e^x + 1)\frac{d^2y}{dx^2} - \frac{dy}{dx} = 0$

(3) $(8e^x + 1)\frac{d^2y}{dx^2} + \frac{dy}{dx} = 0$

(4) $(8e^x - 1)\frac{d^2y}{dx^2} - \frac{dy}{dx} = 0$

38. Let y = y(x) be the solution of the differential equation $(1 + x^2)\frac{dy}{dx} + y = e^{\tan^{-1} x}$, y(1) = 0. Then y(0) is

(1) $\frac{1}{4}(e^{\pi/2} - 1)$ (2) $\frac{1}{2}(1 - e^{\pi/2})$

(3) $\frac{1}{4}(1 - e^{\pi/2})$ (4) $\frac{1}{2}(e^{\pi/2} - 1)$

39. Let y = y(x) be the solution of the differential equation $(2x \log_e x)\frac{dy}{dx} + 2y = \frac{3}{x}\log_e x$, x > 0 and $y(e^{-1}) = 0$. Then, y(e) is equal to

(1) $-\frac{3}{2e}$ (2) $-\frac{2}{3e}$

(3) $-\frac{3}{e}$ (4) $-\frac{2}{e}$

40. Let a conic C pass through the point (4, –2) and P(x, y), $x \geq 3$, be any point on C. Let the slope of the line touching the conic C only at a single point P be half the slope of the line joining the points P and (3, –5). If the focal distance of the point (7, 1) on C is d, then 12d equals_______.

41. Let y = y(x) be the solution of the differential equation $(1 + y^2)e^{\tan x}dx + \cos^2 x(1 + e^{2\tan x})dy = 0$, y(0) = 1. Then $y\left(\frac{\pi}{4}\right)$ is equal to :

(1) $\frac{2}{e}$ (2) $\frac{1}{e^2}$

(3) $\frac{1}{e}$ (4) $\frac{2}{e^2}$

SOLUTIONS

1. Ans. (4)

Sol. $\frac{dx}{dt}+ax=0$

$\frac{dx}{x}=-adt$

$\int\frac{dx}{x}=-a\int dt$

$\ln|x|=-at+c$

at t = 0, x = 2

$\ln 2=0+c$

$\ln x=-at+\ln 2$

$\frac{x}{2}=e^{-at}$

$x=2e^{-at}$ ….(i)

$\frac{dy}{dt}+by=0$

$\frac{dy}{y}=-bdt$

$\ln|y|=-bt+\lambda$

t = 0, y = 1

$0=0+\lambda$

$y=e^{-bt}$ ….(ii)

According to question

$3y(1)=2x(1)$

$3e^{-b}=2(2e^{-a})$

$e^{a-b}=\frac{4}{3}$

For x(t) = y(t)

$\Rightarrow 2e^{-at}=e^{-bt}$

$2=e^{(a-b)t}$

$2=\left(\frac{4}{3}\right)^{t}$

$\log_{\frac{4}{3}}2=t$

2. Ans. (29)

Sol. $2x+3y-2=t$ $\quad$ $4x+6y-4=2t$

$2+3\frac{dy}{dx}=\frac{dt}{dx}$ $\quad$ $4x+6y-7=2t-3$

$\frac{dy}{dx}=\frac{-(2x+3y-2)}{4x+6y-7}$

$\frac{dt}{dx}=\frac{-3t+4t-6}{2t-3}=\frac{t-6}{2t-3}$

$\int\frac{2t-3}{t-6}dt=\int dx$

$\int\left(\frac{2t-12}{t-6}+\frac{9}{t-6}\right).dt=x$

$2t+9\ln(t-6)=x+c$

$2(2x+3y-2)+9\ln(2x+3y-8)=x+c$

$x=0, y=3$

$c=14$

$4x+6y-4+9\ln(2x+3y-8)=x+14$

$x+2y+3\ln(2x+3y-8)=6$

$\alpha=1, \beta=2, \gamma=8$

$\alpha+2\beta+3\gamma=1+4+24=29$

3. Ans. (1)

Sol. $(x^2-4)dy-(y^2-3y)dx=0$

$\Rightarrow\int\frac{dy}{y^2-3y}=\int\frac{dx}{x^2-4}$

$\Rightarrow\frac{1}{3}\int\frac{y-(y-3)}{y(y-3)}dy=\int\frac{dx}{x^2-4}$

$\Rightarrow\frac{1}{3}(\ln|y-3|-\ln|y|)=\frac{1}{4}\ln\left|\frac{x-2}{x+2}\right|+C$

$\Rightarrow\frac{1}{3}\ln\left|\frac{y-3}{y}\right|=\frac{1}{4}\ln\left|\frac{x-2}{x+2}\right|+C$

At $x=4, y=\frac{3}{2}$

$\therefore C=\frac{1}{4}\ln 3$

$\therefore\frac{1}{3}\ln\left|\frac{y-3}{y}\right|=\frac{1}{4}\ln\left|\frac{x-2}{x+2}\right|+\frac{1}{4}\ln(3)$

At x = 10

$\frac{1}{3}\ln\left|\frac{y-3}{y}\right|=\frac{1}{4}\ln\left|\frac{2}{3}\right|+\frac{1}{4}\ln(3)$

$\ln\left|\frac{y-3}{y}\right| = \ln 2^{3/4}$, $\forall x > 2, \frac{dy}{dx} < 0$

as $y(4) = \frac{3}{2} \Rightarrow y \in (0, 3)$

$-y + 3 = 8^{1/4}.y$

$y = \frac{3}{1+8^{1/4}}$

4. Ans. (11)

Sol. $\frac{dy}{dx} = \frac{x+y-2}{x-y}$

$x = X + h, y = Y + k$

$\frac{dY}{dX} = \frac{X+Y}{X-Y}$

$\left.\begin{array}{l} h+k-2=0 \\ h-k=0 \end{array}\right\} h = k = 1$

$Y = vX$

$v + X\frac{dv}{dX} = \frac{1+v}{1-v} \Rightarrow X\frac{dv}{dX} = \frac{1+v^2}{1-v}$

$\frac{1-v}{1+v^2}dv = \frac{dX}{X}$

$\tan^{-1} v - \frac{1}{2}\ln(1+v^2) = \ln|X| + C$

As curve is passing through (2, 1)

$\tan^{-1}\left(\frac{y-1}{x-1}\right) - \frac{1}{2}\ln\left(1+\left(\frac{y-1}{x-1}\right)^2\right) = \ln|x-1|$

$\therefore \alpha = 1$ and $\beta = 2$

$\Rightarrow 5\beta + \alpha = 11$

5. Ans. (1)

Sol. $\frac{dy}{dx} - \left(\frac{\sin 2x}{1+\cos^2 x}\right)y = \sin x$

I.F. = $1 + \cos^2 x$

$y \cdot (1+\cos^2 x) = \int(\sin x)dx$

$= -\cos x + C$

$x = 0, C = 1$

$y\left(\frac{\pi}{2}\right) = 1$

6. Ans. (3)

Sol. $\int\left(\frac{1}{x} + \frac{\ln x}{x}\right)dx + \int\frac{dy}{1+y^2} = 0$

$\ln x + \frac{(\ln x)^2}{2} + \tan^{-1} y = C$

Put x = y = 1

$\therefore C = \frac{\pi}{4}$

$\Rightarrow \ln x + \frac{(\ln x)^2}{2} + \tan^{-1} y = \frac{\pi}{4}$

Put x = e

$\Rightarrow y = \tan\left(\frac{\pi}{4} - \frac{3}{2}\right) = \frac{1-\tan\frac{3}{2}}{1+\tan\frac{3}{2}}$

$\therefore \alpha = 1, \beta = 1$

$\Rightarrow \alpha + 2\beta = 3$

7. Ans. (1)

Sol. Differential equation :–

$x\cos\frac{y}{x}\frac{dy}{dx} = y\cos\frac{y}{x} + x$

$\cos\frac{y}{x}\left[x\frac{dy}{dx} - y\right] = x$

Divide both sides by x^2

$\cos\frac{y}{x}\left(\frac{x\frac{dy}{dx} - y}{x^2}\right) = \frac{1}{x}$

Let $\frac{y}{x} = t$

$\cos t\left(\frac{dt}{dx}\right) = \frac{1}{x}$

$\cos t\, dt = \frac{1}{x}dx$

Integrating both sides

$\sin t = \ln|x| + c$

$\sin\frac{y}{x} = \ln|x| + c$

Using $y(1) = \frac{\pi}{3}$, we get $c = \frac{\sqrt{3}}{2}$

So, $\alpha = \sqrt{3} \Rightarrow \alpha^2 = 3$

8. Ans. (2)

Sol. $f(1) = 1,\ f(a) = 0$

$$f^2(x) = \lim_{r\to x}\left(\frac{2r^2(f^2(r) - f(x)f(r))}{r^2 - x^2} - r^3 e^{\frac{f(r)}{r}}\right)$$

$$= \lim_{r\to x}\left(\frac{2r^2 f(r)}{r+x}\frac{(f(r) - f(x))}{r - x} - r^3 e^{\frac{f(r)}{r}}\right)$$

$$f^2(x) = \frac{2x^2 f(x)}{2x} f'(x) - x^3 e^{\frac{f(x)}{x}}$$

$$y^2 = xy\frac{dy}{dx} - x^3 e^{\frac{y}{x}}$$

$$\frac{y}{x} = \frac{dy}{dx} - \frac{x^2}{y} e^{\frac{y}{x}}$$

Put $\boxed{y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}}$

$$v = v + x\frac{dv}{dx} - \frac{x}{v}e^v$$

$$\frac{dv}{dx} = \frac{e^v}{v} \Rightarrow e^{-v}\, v dv = dx$$

Integrating both side

$e^v(x + c) + 1 + v = 0$

$f(1) = 1 \Rightarrow x = 1,\ y = 1$

$$\Rightarrow c = -1 - \frac{2}{e}$$

$$e^v\left(-1 - \frac{2}{e} + x\right) + 1 + v = 0$$

$$e^{\frac{y}{x}}\left(-1 - \frac{2}{e} + x\right) + 1 + \frac{y}{x} = 0$$

$x = a,\ y = 0 \Rightarrow a = \frac{2}{e}$

$ae = 2$

9. Ans. (1)

Sol. $\frac{dy}{dx} = 2(x - 1)\sin x + (x^2 - 2x)\cos x$

Now both side integrate

$$y(x) = \int 2(x-1)\sin x\, dx + \left[(x^2 - 2x)(\sin x) - \int (2x - 2)\sin x\, dx\right]$$

$$y(x) = (x^2 - 2x)\sin x + \lambda$$

$$y(0) = 0 + \lambda \Rightarrow 2 = \lambda$$

$$y(x) = (x^2 - 2x)\sin x + 2$$

$$y(2) = 2$$

10. Ans. (97)

Sol. $\frac{dy}{dx} - \frac{xy}{1 - x^2} = \frac{(x^3 + 2)\sqrt{3(1 - x^2)}}{1 - x^2}$

$$\text{IF} = e^{-\int \frac{x}{1-x^2}dx} = e^{+\frac{1}{2}\ln(1-x^2)} = \sqrt{1 - x^2}$$

$$y\sqrt{1 - x^2} = \sqrt{3}\int (x^3 + 2)\, dx$$

$$y\sqrt{1 - x^2} = \sqrt{3}\left(\frac{x^4}{4} + 2x\right) + c$$

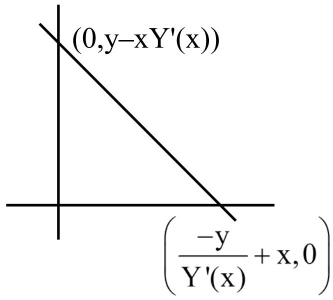
$\Rightarrow y(0) = 0 \qquad \therefore c = 0$

$$y\left(\frac{1}{2}\right) = \frac{65}{32} = \frac{m}{n}$$

$m + n = 97$

11. Ans. (20)

Sol. $A = \frac{1}{2}\left(\frac{-y}{Y'(x)} + x\right)(y - xY'(x)) = \frac{-y^2}{2Y'(x)} + 1$



$$\Rightarrow (-y + xY'(x))(y - xY'(x)) = -y^2 + 2Y'(x)$$

$$-y^2 + xyY'(x) + xyY'(x) - x^2[Y'(x)]^2 = -y^2 + 2Y'(x)$$

$2xy - x^2\, Y'(x) = 2$

$$\frac{dy}{dx} = \frac{2xy - 2}{x^2}$$

$$\frac{dy}{dx} - \frac{2}{x}y = \frac{-2}{x^2}$$

$$\text{I.F.} = e^{-2\ln x} = \frac{1}{x^2}$$

$$y\cdot\frac{1}{x^2} = \frac{2}{3}x^{-3} + c$$

Put x = 1, y = 1

$$1 = \frac{2}{3} + c \Rightarrow c = \frac{1}{3}$$

$$Y = \frac{2}{3}\cdot\frac{1}{X} + \frac{1}{3}X^2$$

$$\Rightarrow 12Y(2) = \frac{5}{3}\cdot 12 = 20$$

12. Ans. (3)

Sol. $\frac{dx}{dy}=\frac{x}{y}\left(\ln\left(\frac{x}{y}\right)+1\right)$

Let $\frac{x}{y}=t \Rightarrow x = ty$

$\frac{dx}{dy}=t+y\frac{dt}{dy}$

$t+y\frac{dt}{dy}=t(\ln(t)+1)$

$y\frac{dt}{dy}=t\ln(t) \Rightarrow \frac{dt}{t\ln(t)}=\frac{dy}{y}$

$\Rightarrow \int\frac{dt}{t.\ln(t)}=\int\frac{dy}{y}$

$\Rightarrow \int\frac{dp}{p}=\int\frac{dy}{y}$ let ln t = p

$\frac{1}{t}dt=dp$

$\Rightarrow$ lnp = lny +c

ln(ln t) = ln y + c

$\ln\left(\ln\left(\frac{x}{y}\right)\right)=\ln y+c$

at x = e, y = 1

$\ln\left(\ln\left(\frac{e}{1}\right)\right)=\ln(1)+c \Rightarrow c = 0$

$\ln\left|\ln\left(\frac{x}{y}\right)\right|=\ln y$

$\left|\ln\left(\frac{x}{y}\right)\right|=e^{\ln y}$

$\left|\ln\left(\frac{x}{y}\right)\right|=y$

13. Ans. (1)

Sol. $\frac{dy}{dx}=\frac{\sin x+y\cos x}{\sin x.\cos x\left(\frac{1}{\cos x}-\sin x.\frac{\sin x}{\cos x}\right)}$

$=\frac{\sin x+y\cos x}{\sin x(1-\sin^2 x)}$

$\frac{dy}{dx}=\sec^2 x+y.2(\operatorname{cosec}2x)$

$\frac{dy}{dx}-2\operatorname{cosec}(2x)y=\sec^2 x$

$\frac{dy}{dx}+p.y=Q$

$I.F.=e^{\int pdx}=e^{\int -2\operatorname{cosec}(2x)dx}$

Let 2x = t

$2\frac{dx}{dt}=1$

$dx=\frac{dt}{2}$

$=e^{-\int \operatorname{cosec}(t)dt}$

$=e^{-\ln\left|\tan\frac{t}{2}\right|}$

$=e^{-\ln|\tan x|}=\frac{1}{|\tan x|}$

$y(IF)=\int Q(IF)dx+c$

$\Rightarrow y\frac{1}{|\tan x|}=\int\sec^2 x.\frac{1}{|\tan x|}+c$

$y.\frac{1}{|\tan x|}=\int\frac{dt}{|t|}+c$ for tan x = t

$y.\frac{1}{|\tan x|}=\ln|t|+c$

$y=|\tan x|(\ln|\tan x|+c)$

Put $x=\frac{\pi}{4}$, y = 2

2 = ln 1 + c $\Rightarrow$ c = 2

$y=|\tan x|(\ln|\tan x|+2)$

$y\left(\frac{\pi}{3}\right)=\sqrt{3}(\ln\sqrt{3}+2)$

14. Ans. (3)

Sol. $\frac{dT}{dt} = -k(T - 80)$

$\int_{160}^{T} \frac{dT}{(T-80)} = \int_{0}^{t} Kdt$

$\left[\ln|T - 80|\right]_{160}^{T} = -kt$

$\ln|T - 80| - \ln 80 = -kt$

$\ln\left|\frac{T-80}{80}\right| = -kt$

$T = 80 + 80e^{-kt}$

$120 = 80 + 80e^{-k.15}$

$\frac{40}{80} = e^{-k15} = \frac{1}{2}$

$\therefore T(45) = 80 + 80e^{-k.45}$

$= 80 + 80\left(e^{-k.15}\right)^3$

$= 80 + 80 \cdot \frac{1}{8}$

$= 90$

15. Ans. (9)

Sol. $\sec^2 x \frac{dx}{dy} + e^{2y} \tan^2 x + \tan x = 0$

$\left(\text{Put } \tan x = t \Rightarrow \sec^2 x \frac{dx}{dy} = \frac{dt}{dy}\right)$

$\frac{dt}{dy} + e^{2y} \cdot t^2 + t = 0$

$\frac{dt}{dy} + t = -t^2.e^{2y}$

$\frac{1}{t^2}\frac{dt}{dy} + \frac{1}{t} = -e^{2y}$

$\left(\text{Put } \frac{1}{t} = u \quad \frac{-1}{t^2}\frac{dt}{dy} = \frac{du}{dy}\right)$

$\frac{-du}{dy} + u = -e^{2y}$

$\frac{du}{dy} - u = e^{2y}$

$\text{I.F.} = e^{-\int dy} = e^{-y}$

$ue^{-y} = \int e^{-y} \cdot e^{2y} dy$

$\frac{1}{\tan x} \cdot e^{-y} = e^{y} + c$

$x = \frac{\pi}{4}, y = 0, c = 0$

$x = \frac{\pi}{6}, \ y = \alpha$

$\sqrt{3}e^{-\alpha} = e^{\alpha} + 0$

$e^{2\alpha} = \sqrt{3}$

$e^{8\alpha} = 9$

16. Ans. (4)

Sol. $\frac{dy}{dx} = 2x(x+y)^3 - x(x+y) - 1$

$x + y = t$

$\frac{dt}{dx} - 1 = 2xt^3 - xt - 1$

$\frac{dt}{2t^3 - t} = xdx$

$\frac{tdt}{2t^4 - t^2} = xdx$

Let $t^2 = z$

$\int \frac{dz}{2(2z^2 - z)} = \int xdx$

$\int \frac{dz}{4z\left(z - \frac{1}{2}\right)} = \int xdx$

$\ln\left|\frac{z - \frac{1}{2}}{z}\right| = x^2 + k$

$z = \frac{1}{2 - \sqrt{e}}$

17. Ans. (14)

Sol. $(t + 1)dx = (2x + (t + 1)^4)dt$

$$\frac{dx}{dt} = \frac{2x + (t+1)^4}{t+1}$$

$$\frac{dx}{dt} - \frac{2x}{t+1} = (t+1)^3$$

$$I.F = e^{-\int \frac{2}{t+1}dt} = e^{-2\ln(t+1)} = \frac{1}{(t+1)^2}$$

$$\frac{x}{(t+1)^2} = \int \frac{1}{(t+1)^2}(t+1)^3 dt + c$$

$$\frac{x}{(t+1)^2} = \frac{(t+1)^2}{2} + c$$

$$\Rightarrow c = \frac{3}{2}$$

$$x = \frac{(t+1)^4}{2} + \frac{3}{2}(t+1)^2$$

put, $t = 1$

$x = 2^3 + 6 = 14$

18. Ans. (Bonus)

Sol. $f(0) = 2, \lim_{x \to -\infty} f(x) = 1$

$f'(x) - \alpha.f(x) = 3$

$I.F = e^{-\alpha x}$

$$y(e^{-\alpha x}) = \int 3.e^{-\alpha x}dx$$

$$f(x).(e^{-\alpha x}) = \frac{3e^{-\alpha x}}{-\alpha} + C$$

$$x = 0 \Rightarrow 2 = \frac{-3}{\alpha} + C \Rightarrow \frac{3}{\alpha} = C - 2 \quad .....(1)$$

$$f(x) = \frac{-3}{\alpha} + C.e^{\alpha x}$$

Case-I $\alpha > 0$

$$x \to -\infty \Rightarrow 1 = \frac{-3}{\alpha} + C(0)$$

$\alpha = -3$ (rejected)

Case-II $\alpha < 0$

as $\lim_{x \to -\infty} f(x) = 1 \Rightarrow C = 0$ and $\frac{-3}{\alpha} = 1 \Rightarrow \alpha = -3$

$\Rightarrow f(x) = 1$ (rejected)

as $f(0) = 2$

$\Rightarrow$ data is inconsistent

19. Ans. (5)

Sol. $$\frac{dx}{dy} - \frac{x}{y} = \frac{1-y^2}{y}$$

Integrating factor $= e^{\int -\frac{1}{y}dy} = \frac{1}{y}$

$$x.\frac{1}{y} = \int \frac{1-y^2}{y^2}dy$$

$$\frac{x}{y} = \frac{-1}{y} - y + c$$

$x = -1 - y^2 + cy$

$x(1) = 1$

$1 = -1 - 1 + c \Rightarrow c = 3$

$x = -1 - y^2 + 3y$

$5x(2) = 5(-1 - 4 + 6)$

$= 5$

20. Ans. (3)

Sol. $$\sec^2 y \frac{dy}{dx} + 2x\sin y \sec y = x^3 \cos y \sec y$$

$$\sec^2 y \frac{dy}{dx} + 2x\tan y = x^3$$

$$\tan y = t \Rightarrow \sec^2 y \frac{dy}{dx} = \frac{dt}{dx}$$

$$\frac{dt}{dx} + 2xt = x^3, \text{ If } = e^{\int 2x\,dx} = e^{x^2}$$

$$te^{x^2} = \int x^3.e^{x^2}dx + c$$

$$x^2 = Z \Rightarrow t.e^Z = \frac{1}{2}\int e^Z.Z\,dZ = \frac{1}{2}[e^Z.Z - e^Z] + c$$

$$2\tan y = (x^2 - 1) + 2ce^{-x^2}$$

$$y(1) = 0 \Rightarrow c = 0 \Rightarrow y(\sqrt{3}) = \frac{\pi}{4}$$

21. Ans. (4)

Sol. $a|x| = |y|\, e^{yx-\beta}$, a, b $\in$ N

$xdy - ydx + xy(xdy + ydx) = 0$

$$\frac{dy}{y} - \frac{dx}{x} + (xdy + ydx) = 0$$

$n|y| - n|x| + xy = c$

$y(1) = 2$

$n|2| - 0 + 2 = c$

$c = 2 + n2$

$n|y| - n|x| + xy = 2 + n2$

$$n|x| = n\left|\frac{y}{2}\right| - 2 + xy$$

$$|x| = \left|\frac{y}{2}\right| e^{xy-2}$$

$2|x| = |y|e^{xy-2}$

$\alpha = 2 \quad \beta = 2 \quad \alpha + \beta = 4$

22. Ans. (1)

Sol. $$\sqrt{1-(y'(x))^2} = y(x)$$

$$1 - \left(\frac{dy}{dx}\right)^2 = y^2$$

$$\left(\frac{dy}{dx}\right)^2 = 1 - y^2$$

$$\frac{dy}{\sqrt{1-y^2}} = dx \text{ OR } \frac{dy}{\sqrt{1-y^2}} = -dx$$

$\Rightarrow \sin^{-1}y = x + c, \sin^{-1}y = -x + c$

$x = 0, y = 0 \quad \Rightarrow \quad c = 0$

$\sin^{-1} y = x$, as $y \geq 0$

$\sin x = y$

$$\Rightarrow \frac{dy}{dx} = \cos x$$

$$\frac{d^2y}{dx^2} = -\sin x$$

$\Rightarrow -\sin x + \sin x + 1 = 1$

23. Ans. (4)

Sol. $$\ln(y) = 3\sin^{-1} x$$

$$\frac{1}{y} . y' = 3\left(\frac{1}{\sqrt{1-x^2}}\right)$$

$$\Rightarrow y' = \frac{3y}{\sqrt{1-x^2}} \text{ at } x = \frac{1}{2}$$

$$\Rightarrow y' = \frac{3e^{3\left(\frac{\pi}{6}\right)}}{\frac{\sqrt{3}}{2}} = 2\sqrt{3}e^{\frac{\pi}{2}}$$

$$\Rightarrow y'' = 3\left(\frac{\sqrt{1-x^2}\,y' - y\frac{1}{2\sqrt{1-x^2}}(-2x)}{(1-x^2)}\right)$$

$$\Rightarrow (1-x^2)y'' = 3\left(3y + \frac{xy}{\sqrt{1-x^2}}\right)$$

$$\downarrow \text{at } x = \frac{1}{2},\; y = e^{3\sin^{-1}\left(\frac{1}{2}\right)} = e^{3\left(\frac{\pi}{6}\right)} = e^{\frac{\pi}{2}}$$

$$(1-x^2)y'\Big|_{\text{at } x=\frac{1}{2}} = 3\left(3e^{\frac{\pi}{2}} + \frac{\frac{1}{2}\left(e^{\frac{\pi}{2}}\right)}{\frac{\sqrt{3}}{2}}\right)$$

$$= 3e^{\frac{\pi}{2}}\left(3 + \frac{1}{\sqrt{3}}\right)$$

$$(1-x^2)y'' - xy\Big|_{\text{at } x=\frac{1}{2}}$$

$$= 3e^{\frac{\pi}{2}}\left(3 + \frac{1}{\sqrt{3}}\right) - \frac{1}{2}\left(2\sqrt{3}e^{\frac{\pi}{2}}\right) = 9e^{\frac{\pi}{2}}$$

24. Ans. (61)

Sol. $\frac{dy}{dx} - 3y = \alpha$

If $= e^{\int -3dx} = e^{-3x}$

$\therefore\ y - e^{-3x} = \int e^{-3x} \cdot \alpha dx$

$y\, e^{-3x} = \frac{\alpha\, e^{-3x}}{-3} + c$

(* e^{3x})

$y = \frac{\alpha}{-3} + C\, e^{3x}$

on substituting x = 0, y = 1

$x \to -\infty$, y = 7

we get $y = 7 - 6e^{3x}$

$9f(-\log_e 3) = 61$

25. Ans. (1)

Sol. $(2y - 5)\frac{dy}{dx} = -3$

$(2y - 5)dy = -3dx$

$2 \cdot \frac{y^2}{2} - 5y = -3x + \lambda$

$\because$ Curve passes through (0, 1)

$\Rightarrow \lambda = -4$

$\because$ Curve will be

$\left(y - \frac{5}{2}\right)^2 = -3\left(x - \frac{3}{4}\right)$

$\therefore$ Vertex of parabola will be $\left(\frac{3}{4}, \frac{5}{2}\right)$

$\because$ 2x + 3y = 9

26. Ans. (1)

Sol. $(x^2 + y^2)\, dx = 5xydy$

$\Rightarrow \frac{dy}{dx} = \frac{x^2 + y^2}{5xy}$

Put y = Vx

$\Rightarrow V + x\frac{dv}{dx} = \frac{1 + V^2}{5V}$

$\Rightarrow \frac{xdv}{dx} = \frac{1 - 4V^2}{5V}$

$\Rightarrow \int \frac{V}{1 - 4V^2} dV = \int \frac{dx}{5x}$

Let $1 - 4V^2 = t$

$\Rightarrow -8V\, dV = dt$

$\Rightarrow \int \frac{dt}{(-8)(t)} = \int \frac{dx}{5x}$

$\Rightarrow \frac{-1}{8}\ln|t| = \frac{1}{5}\ln|x| + \ln C$

$\Rightarrow -5\ln|t| = 8\ln|x| + \ln K$

$\Rightarrow \ln x^8 + \ln|t^5| + \ln K = 0$

$\Rightarrow x^8 |t^5| = C$

$\Rightarrow x^8 |1 - 4V^2|^5 = C$

$\Rightarrow x^8 \left|\frac{x^2 - 4y^2}{x^2}\right|^5 = C$

$\Rightarrow |x^2 - 4y^2|^5 = Cx^2$

given y(1) = 0

$\Rightarrow |1|^5 = C \Rightarrow C = 1$

$\Rightarrow |x^2 - 4y^2|^5 = x^2$

27. Ans. (3)

Sol. $\int dy = \int \frac{(2x^2 + 2x + 3)}{x^4 + 2x^3 + 3x^2 + 2x + 2} dx$

$y = \int \frac{(2x^2 + 2x + 3)}{(x^2 + 1)(x^2 + 2x + 2)} dx$

$y = \int \frac{dx}{x^2 + 2x + 2} + \int \frac{dx}{x^2 + 1}$

$y = \tan^{-1}(x + 1) + \tan^{-1}x + C$

$y(-1) = \frac{-\pi}{4}$

$\frac{-\pi}{4} = 0 - \frac{\pi}{4} + C \Rightarrow C = 0$

$\Rightarrow y = \tan^{-1}(x + 1) + \tan^{-1}x$

$y(0) = \tan^{-1}1 = \frac{\pi}{4}$

28. Ans. (7)

Sol. $ye^{-x} = \int\left(e^{-x} + 4e^{-x}\sin x\right)dx$

$ye^{-x} = -e^{-x} - 2(e^{-x}\sin x \;\; e^{-x}\cos x) + C$

$y = -1 - 2(\sin x + \cos x) + ce^{x}$

$\because\ y(\pi) = 1 \Rightarrow c = 0$

$y(\pi/2) = -1 - 2 = -3$

Ans = 10 – 3 = 7

29. Ans. (3)

Sol. $C \equiv x^2 + y^2 + gx + gy = 0$ .....(1)

$2x + 2yy' + g + gy' = 0$

$g = -\left(\frac{2x + 2yy'}{1 + y'}\right)$

Put in (1)

$x^2 + y^2 - \left(\frac{2x + 2yy'}{1 + y'}\right)(x + y) = 0$

$(x^2 - y^2 - 2xy)y' = x^2 - y^2 + 2xy$

30. Ans. (18)

Sol. $\text{IF} = e^{\int \frac{2x}{(1+x^2)^2}dx} = e^{\frac{-1}{1+x^2}}$

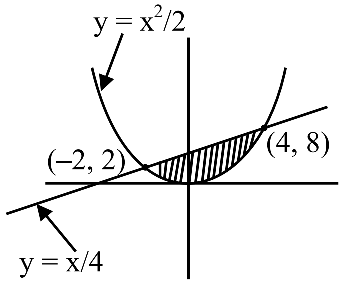
$y\,e^{\frac{-1}{1+x^2}} = \int x\,e^{\frac{1}{1+x^2}} . e^{\frac{-1}{1+x^2}}dx$

$y\,e^{\frac{-1}{1+x^2}} = \frac{x^2}{2} + c$

$(0, 0) \Rightarrow \boxed{C = 0}$

$y(x) = \frac{x^2}{2}e^{\frac{1}{1+x^2}}$

$f(x) = \frac{x^2}{2}$



$A = \int_{-2}^{4}(x + 4) - \frac{x^2}{2}dx = 18$

31. Ans. (4)

Sol. $\frac{dy}{dx} + y\left(\frac{2x^3 + 8x}{\left(x^2 + 4\right)^2}\right) = \frac{2}{\left(x^2 + 4\right)^2}$

$\frac{dy}{dx} + y\left(\frac{2x}{x^2 + 4}\right) = \frac{2}{\left(x^2 + 4\right)^2}$

$\text{IF} = e^{\int \frac{2x}{x^2+4}dx}$

$\text{IF} = x^2 + 4$

$y \times (x^2 + 4) = \int \frac{2}{\left(x^2 + 4\right)^2} \cdot \left(x^2 + 4\right)$

$y(x^2 + 4) = 2\int \frac{dx}{x^2 + 2^2}$

$y\left(x^2 + 4\right) = \frac{2}{2}\tan^{-1}\left(\frac{x}{2}\right) + c$

$0 = 0 + c = c = 0$

$y(x^2 + 4) = \tan^{-1}\left(\frac{x}{2}\right)$

y at x = 2

$y(4 + 4) = \tan^{-1}(1)$

$\boxed{y(2) = \frac{\pi}{32}}$

Option (4) is correct

32. Ans. (31)

Sol. $\frac{dy}{dx} = (x + y + 2)^2$ ...(1), $y(0) = -2$

Let $x + y + 2 = v$

$1 + \frac{dy}{dx} = \frac{dv}{dx}$

from (1) $\frac{dv}{dx} = 1 + v^2$

$\int \frac{dv}{1 + v^2} = \int dx$

$\tan^{-1}(v) = x + C$

$\tan^{-1}(x + y + 2) = x + C$

at $x = 0$ $y = -2 \Rightarrow C = 0$

$\Rightarrow \tan^{-1}(x + y + 2) = x$

$y = \tan x - x - 2$

$f(x) = \tan x - x - 2, x \in \left[0, \frac{\pi}{3}\right]$

$f'(x) = \sec^2 x - 1 > 0 \Rightarrow f(x) \uparrow$

$f_{min} = f(0) = -2 = \beta$

$f_{max} = f\left(\frac{\pi}{3}\right) = \sqrt{3} - \frac{\pi}{3} - 2 = \alpha$

now $(3\alpha + \pi)^2 + \beta^2 = \gamma + \delta\sqrt{3}$

$\Rightarrow (3\alpha + \pi)^2 + \beta^2 = (3\sqrt{3} - 6)^2 + 4$

$\gamma + \delta\sqrt{3} = 67 - 36\sqrt{3}$

$\Rightarrow \gamma = 67$ and $\delta = -36 \Rightarrow \gamma + \delta = 31$

33. Ans. (2)

Sol. $\frac{dy}{dx} + 2y = \sin 2x$, $y(0) = \frac{3}{4}$

$I.F = e^{\int 2dx} = e^{2x}$

$y.e^{2x} = \int e^{2x} \sin 2x\, dx$

$y.e^{2x} = \frac{e^{2x}(2\sin 2x - 2\cos 2x)}{4+4} + C$

$x = 0, y = \frac{3}{4} \Rightarrow \frac{3}{4}.1 = \frac{1(0-2)}{8} + C$

$\frac{3}{4} = -\frac{1}{4} + C$

$1 = C$

$y = \frac{2\sin 2x - 2\cos 2x}{8} + 1.e^{-2x}$

$x = \frac{\pi}{8}$, $y = \frac{1}{8}\left(2\sin\frac{\pi}{4} - 2\cos\frac{\pi}{4}\right) + e^{-2\left(\frac{\pi}{8}\right)}$

$y = 0 + e^{-\frac{\pi}{4}}$

34. Ans. (3)

Sol. $\frac{dy}{dx} = \frac{(2+\alpha)x - \beta y + 2}{\beta x - y(2\alpha + \beta) + 4\alpha}$

$\beta xdy - (2\alpha + \beta)ydy + 4\alpha dy = (2+\alpha)xdx - \beta ydx + 2dx$

$\beta(xdy + ydx) - (2\alpha + \beta)ydy + 4\alpha dy = (2+\alpha)xdx + 2dx$

$\beta xy - \frac{(2\alpha + \beta)y^2}{2} + 4\alpha y = \frac{(2+\alpha)x^2}{2}$

$\Rightarrow \beta = 0$ for this to be circle

$(2+\alpha)\frac{x^2}{2} + \alpha y^2 + 2x - 4\alpha y = 0$

coeff. of $x^2 = y^2$ $\Rightarrow$ $2 + a = 2a$

$\Rightarrow \boxed{\alpha = 2}$

i.e. $2x^2 + 2y^2 + 2x - 8y = 0$

$x^2 + y^2 + x - 4y = 0$

$rd = \sqrt{\frac{1}{4} + 4} = \frac{\sqrt{17}}{2}$

35. Ans. (3)

Sol. $(e^y + 1)\cos x\, dx + e^y \sin x\, dy = 0$

$\Rightarrow d\left(\left(e^y + 1\right)\sin x\right) = 0$

$\left(e^y + 1\right)\sin x = C$

It passes through $\left(\frac{\pi}{2}, 0\right)$

$\Rightarrow c = 2$

Now, $x = \frac{\pi}{6}$

$\Rightarrow e^y = 3$

36. Ans. (24)

Sol. $\lim\limits_{t\to x}\dfrac{t^2f(x)-x^2f(t)}{t-x}=1$

$\lim\limits_{t\to x}\dfrac{2t.f(x)-x^2f'(x)}{1}=1$

2x.f(x) – x2f′(x) = 1

$\dfrac{dy}{dx}-\dfrac{2}{x}\cdot y=\dfrac{-1}{x^2}$

$I.f.=e^{\int-\frac{2}{x}dx}=\dfrac{1}{x^2}$

$\therefore\ \dfrac{y}{x^2}=\int-\dfrac{1}{x^4}dx+C$

$\dfrac{y}{x^2}=\dfrac{1}{3x^3}+C$

Put f(1) = 1

$C=\dfrac{2}{3}$

$y=\dfrac{1}{3x}+\dfrac{2x^2}{3}$

$y=\dfrac{2x^3+1}{3x}$

$f(2)=\dfrac{17}{6}$

$f(3)=\dfrac{55}{9}$

$2f(2)+3f(3)=\dfrac{17}{3}+\dfrac{55}{3}=\dfrac{72}{3}=24$

37. Ans. (3)

Sol. $\int_0^a f(x)dx=e^{-a}+4a^2+a-1$

$f(a)=-e^{-a}+8a+1$

$f(x)=-e^{-x}+8x+1$

Now $y=C_1f(x)+C_2$

$\dfrac{dy}{dx}=C_1f'(x)=C_1(e^{-x}+8)$ …..(1)

$\dfrac{d^2y}{dx^2}=-C_1e^{-x}\ \Rightarrow\ -e^{x}\dfrac{d^2y}{dx^2}$

Put in equation (1)

$\dfrac{dy}{dx}=-e^{x}\dfrac{d^2y}{dx^2}(e^{-x}+8)$

$(8e^{x}+1)\dfrac{d^2y}{dx^2}+\dfrac{dy}{dx}=0$

38. Ans. (2)

Sol. $\dfrac{dy}{dx}+\dfrac{y}{1+x^2}=\dfrac{e^{\tan^{-1}x}}{1+x^2}$

$I.F.=e^{\int\frac{1}{1+x^2}dx}=e^{\tan^{-1}x}$

$y.e^{\tan^{-1}x}=\int\left(\dfrac{e^{\tan^{-1}x}}{1+x^2}\right)e^{\tan^{-1}x}.dx$

Let $\tan^{-1}x=z$ $\quad\therefore\ \dfrac{dx}{1+x^2}=dz$

$\therefore\ y.e^{z}=\int e^{2z}dz=\dfrac{e^{2z}}{2}+C$

$y.e^{\tan^{-1}x}=\dfrac{e^{2\tan^{-1}x}}{2}+C$

$\Rightarrow y=\dfrac{e^{\tan^{-1}x}}{2}+\dfrac{C}{e^{\tan^{-1}x}}$

$\because\ y(1)=0\ \Rightarrow 0=\dfrac{e^{\pi/4}}{2}+\dfrac{C}{e^{\pi/4}}\ \Rightarrow C=\dfrac{-e^{\pi/2}}{2}$

$\therefore\ y=\dfrac{e^{\tan^{-1}x}}{2}-\dfrac{e^{\pi/2}}{2e^{\tan^{-1}x}}$

$\boxed{\therefore y(0)=\dfrac{1-e^{\pi/2}}{2}}$

39. Ans. (3)

Sol. $\dfrac{dy}{dx}+\dfrac{y}{x\ \ell n\,x}=\dfrac{3}{2x^2}$

$\therefore\ I.F.=e^{\int\frac{1}{x\ \ell n\,x}dx}=e^{\ell n(\ell n(x))}=\ell n\,x$

$\therefore\ y\ \ell n\,x=\int\dfrac{3\,\ell n\,x}{2x^2}dx$

$=\dfrac{3\,\ell n\,x}{2}\int x^{-2}dx-\int\left(\dfrac{3}{2x}.\int x^{-2}dx\right)dx$

$=\dfrac{3\,\ell n\,x}{2}\left(-\dfrac{1}{x}\right)-\int\dfrac{3}{2x}\left(-\dfrac{1}{x}\right)dx$

$y.\ \ell n\,x=\dfrac{-3\,\ell n\,x}{2x}-\dfrac{3}{2x}+C$

$\because\ y(e^{-1})=0$

$\therefore\ 0\,(-1)=\dfrac{3e}{2}-\dfrac{3e}{2}+C\quad\Rightarrow\ C=0$

$\therefore\ y=\dfrac{-3\,\ell n\,x}{2x}-\dfrac{3}{2x}$

$\boxed{\therefore y(e)=\dfrac{-3}{2e}-\dfrac{3}{2e}=\dfrac{-3}{e}}$

40. Ans. (75)

Sol. P(x, y) & $x \geq 3$

Slope of line at P(x, y) will be $\frac{dy}{dx} = \frac{1}{2}\left(\frac{y+5}{x-3}\right)$

$\Rightarrow 2\frac{dy}{(y+5)} = \frac{1}{(x-3)}dx$

$\Rightarrow 2\ n(y + 5) = \ n(x - 3) + C$

Passes through (4, –2)

$\Rightarrow 2\ n(3) = \ n(1) + C$

$\Rightarrow C = 2\ n(3)$

$\Rightarrow 2\ n(y + 5) = \ n(x - 3) + 2\ n(3)$

$\Rightarrow 2\left(\ n\left(\frac{y+5}{3}\right)\right) = \ n(x-3)$

$\Rightarrow \left(\frac{y+5}{3}\right)^2 = (x-3)$

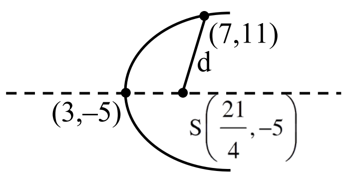
$\Rightarrow (y + 5)^2 = 9(x - 3)$

↓

Parabola

4a = 9

$a = \frac{9}{4}$



$d = \sqrt{\left(\frac{7}{4}\right)^2 + 6^2}$

$d = \frac{\sqrt{625}}{4}$

$d = \frac{25}{4}$

12d = 75

41. Ans. (3)

Sol. $(1 + y^2)\, e^{\tan x}dx + \cos^2 x(1 + e^{2\tan x})\, dy = 0$

$\int \frac{\sec^2 x\, e^{\tan x}}{1 + e^{2\tan x}}dx + \int \frac{dy}{1+y^2} = C$

$\Rightarrow \tan^{-1}(e^{\tan x}) + \tan^{-1} y = C$

for x = 0, y = 1, $\tan^{-1}(1) + \tan^{-1} 1 = C$

$C = \frac{\pi}{2}$

$\tan^{-1}(e^{\tan x}) + \tan^{-1} y = \frac{\pi}{2}$

Put $x = \pi$, $\tan^{-1} e + \tan^{-1} y = \frac{\pi}{2}$

$\tan^{-1} y = \cot^{-1} e$

$y = \frac{1}{e}$