FUNCTIONS

1. The function f : N – {1} → N; defined by f(n) = the highest prime factor of n, is :

(1) both one-one and onto

(2) one-one only

(3) onto only

(4) neither one-one nor onto

2. Let $f:R-\left\{\frac{-1}{2}\right\}\to R$ and $g:R-\left\{\frac{-5}{2}\right\}\to R$ be defined as $f(x)=\frac{2x+3}{2x+1}$ and $g(x)=\frac{|x|+1}{2x+5}$.

Then the domain of the function fog is :

(1) $R-\left\{-\frac{5}{2}\right\}$

(2) R

(3) $R-\left\{-\frac{7}{4}\right\}$

(4) $R-\left\{-\frac{5}{2},-\frac{7}{4}\right\}$

3. If $f(x)=\begin{cases}2+2x, & -1\le x<0\\ 1-\frac{x}{3}, & 0\le x\le 3\end{cases}$;

$g(x)=\begin{cases}-x, & -3\le x\le 0\\ x, & 0<x\le 1\end{cases}$, then range of (fog(x)) is

(1) (0, 1] (2) [0, 3)

(3) [0, 1] (4) [0, 1)

4. Let the set $C=\left\{(x,y)\mid x^2-2^y=2023, x,y\in\mathbb{N}\right\}$. Then $\sum_{(x,y)\in C}(x+y)$ is equal to _______.

5. Let A = {1, 2, 3,….7} and let P(A) denote the power set of A. If the number of functions $f:A\to P(A)$ such that $a\in f(a), \forall a\in A$ is m^n, m and $n\in N$ and m is least, then m + n is equal to _______.

6. If $f(x)=\frac{4x+3}{6x-4}, x\ne\frac{2}{3}$ and (fof) (x) = g(x), where $g:\mathbb{R}-\left\{\frac{2}{3}\right\}\to\mathbb{R}-\left\{\frac{2}{3}\right\}$, then (gogog) (4) is equal to

(1) $-\frac{19}{20}$ (2) $\frac{19}{20}$ (3) –4 (4) 4

7. If the function $f:(-\infty,-1]\to(a,b]$ defined by $f(x)=e^{x^3-3x+1}$ is one-one and onto, then the distance of the point P(2b + 4, a + 2) from the line $x+e^{-3}y=4$ is :

(1) $2\sqrt{1+e^6}$ (2) $4\sqrt{1+e^6}$

(3) $3\sqrt{1+e^6}$ (4) $\sqrt{1+e^6}$

8. Let f : **R** → **R** and g : **R** → **R** be defined as

$f(x)=\begin{cases}\log_e x, & x>0\\ e^{-x}, & x\le 0\end{cases}$ and

$g(x)=\begin{cases}x, & x\ge 0\\ e^{x}, & x<0\end{cases}$. Then, gof : **R** → **R** is :

(1) one-one but not onto

(2) neither one-one nor onto

(3) onto but not one-one

(4) both one-one and onto

9. If the domain of the function

$f(x)=\frac{\sqrt{x^2-25}}{(4-x^2)}+\log_{10}(x^2+2x-15)$

is $(-\infty,\alpha)\cup[\beta,\infty)$, then $\alpha^2+\beta^3$ is equal to :

(1) 140 (2) 175

(3) 150 (4) 125

10. Let $f(x)=\begin{cases}-a & \text{if } -a\le x\le 0\\ x+a & \text{if } 0<x\le a\end{cases}$

where a > 0 and $g(x)=(f|x|)-|f(x)|)/2$.

Then the function g : [–a, a] → [–a, a] is

(1) neither one-one nor onto.

(2) both one-one and onto.

(3) one-one.

(4) onto

11. Let $A = \{(x, y) : 2x + 3y = 23, x, y \in N\}$ and $B = \{x : (x, y) \in A\}$. Then the number of one-one functions from A to B is equal to _____.

12. If a function f satisfies $f(m + n) = f(m) + f(n)$ for all $m, n \in N$ and $f(1) = 1$, then the largest natural number λ such that

$$\sum_{k=1}^{2022} f(\lambda + k) \le (2022)^2$$ is equal to ________.

13. Let $f, g : R \to R$ be defined as : $f(x) = |x - 1|$ and $g(x) = \begin{cases} e^x, & x \ge 0 \\ x+1, & x \le 0 \end{cases}$. Then the function $f(g(x))$ is

(1) neither one-one nor onto.

(2) one-one but not onto.

(3) both one-one and onto.

(4) onto but not one-one.

14. Consider the function $f : \mathbb{R} \to \mathbb{R}$ defined by

$f(x) = \dfrac{2x}{\sqrt{1+9x^2}}$. If the composition of

$f,\underbrace{(f \text{ o } f \text{ o } f \text{ o...o } f)}_{10 \text{times}}(x) = \dfrac{2^{10}x}{\sqrt{1+9\alpha x^2}}$, then the value of $\sqrt{3\alpha+1}$ is equal to …..

15. Let $A = \{1, 3, 7, 9, 11\}$ and $B = \{2, 4, 5, 7, 8, 10, 12\}$. Then the total number of one-one maps $f : A \to B$, such that $f(1) + f(3) = 14$, is :

(1) 180

(2) 120

(3) 480

(4) 240

16. If $S = \{a \in R : |2a - 1| = 3[a]+2\{a\}\}$, where [t] denotes the greatest integer less than or equal to t and {t} represents the fractional part of t, then $72\sum_{a \in S} a$ is equal to __________.

17. Let $f(x) = \dfrac{1}{7 - \sin 5x}$ be a function defined on R. Then the range of the function f(x) is equal to:

(1) $\left[\frac{1}{8}, \frac{1}{5}\right]$

(2) $\left[\frac{1}{7}, \frac{1}{6}\right]$

(3) $\left[\frac{1}{7}, \frac{1}{5}\right]$

(4) $\left[\frac{1}{8}, \frac{1}{6}\right]$

18. The function $f(x) = \dfrac{x^2 + 2x - 15}{x^2 - 4x + 9}$, $x \in R$ is

(1) both one-one and onto.

(2) onto but not one-one.

(3) neither one-one nor onto.

(4) one-one but not onto.

19. Let [t] be the greatest integer less than or equal to t. Let A be the set of all prime factors of 2310 and $f : A \to \mathbb{Z}$ be the function $f(x) = \left[\log_2\left(x^2 + \left[\frac{x^3}{5}\right]\right)\right]$.

The number of one-to-one functions from A to the range of f is :

(1) 20

(2) 120

(3) 25

(4) 24

SOLUTIONS

1. **Ans. (4)**

Sol. $f : N - \{1\} \to N$

f(n) = The highest prime factor of n.

$f(2) = 2$

$f(4) = 2$

$\Rightarrow$ many one

4 is not image of any element

$\Rightarrow$ into

Hence many one and into

Neither one-one nor onto.

2. **Ans. (1)**

Sol. $f(x) = \frac{2x+3}{2x+1}; x \neq -\frac{1}{2}$

$g(x) = \frac{|x|+1}{2x+5}, x \neq -\frac{5}{2}$

Domain of f(g(x))

$f(g(x)) = \frac{2g(x)+3}{2g(x)+1}$

$x \neq -\frac{5}{2}$ and $\frac{|x|+1}{2x+5} \neq -\frac{1}{2}$

$x \in R - \left\{-\frac{5}{2}\right\}$ and $x \in R$

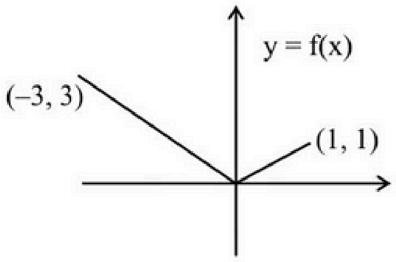
$\therefore$ Domain will be $R - \left\{-\frac{5}{2}\right\}$

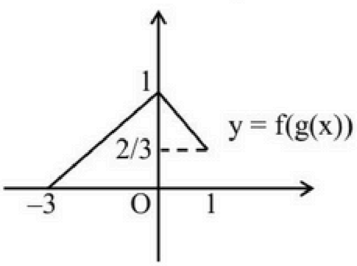
3. **Ans. (3)**

Sol. $f(g(x)) = \begin{cases} 2 + 2g(x) & , \quad -1 \leq g(x) < 0 \quad .....(1) \\ 1 - \frac{g(x)}{3} & , \quad 0 \leq g(x) \leq 3 \quad .....(2) \end{cases}$

By (1) $x \in \phi$

And by (2) $x \in [-3, 0]$ and $x \in [0, 1]$





Range of f(g(x)) is [0, 1]

4. **Ans. (46)**

Sol. $x^2 - 2^y = 2023$

$\Rightarrow \boxed{x = 45, y = 1}$

$\sum_{(x,y)\in C} (x + y) = 46.$

5. **Ans. (44)**

Sol. $f : A \to P(A)$

$a \in f(a)$

That means 'a' will connect with subset which contain element 'a'.

Total options for 1 will be 2^6. (Because 2^6 subsets contains 1)

Similarly, for every other element

Hence, total is $2^6 \times 2^6 \times 2^6 \times 2^6 \times 2^6 \times 2^6 \times 2^6 = 2^{42}$

Ans. 2 + 42 = 44

6. **Ans. (4)**

Sol. $f(x) = \frac{4x+3}{6x-4}$

$g(x) = \frac{4\left(\frac{4x+3}{6x-4}\right)+3}{6\left(\frac{4x+3}{6x-4}\right)-4} = \frac{34x}{34} = x$

$g(x) = x \therefore g(g(g(4))) = 4$

7. **Ans. (1)**

Sol. $f(x) = e^{x^3-3x+1}$

$f'(x) = e^{x^3-3x+1}.(3x^2 - 3)$

$= e^{x^3-3x+1}.3(x-1)(x+1)$

For $f'(x) \geq 0$

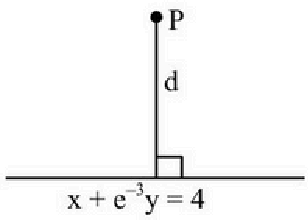
$\therefore f(x)$ is increasing function

$\therefore a = e^{-\infty} = 0 = f(-\infty)$

$b = e^{-1+3+1} = e^3 = f(-1)$

P(2b + 4, a + 2)

$\therefore P(2e^3 + 4, 2)$

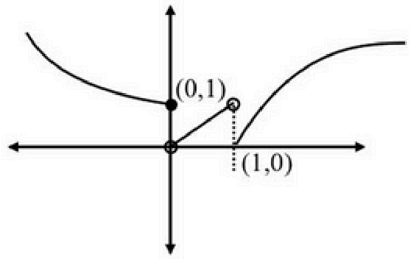


$d = \frac{(2e^3 + 4) + 2e^{-3} - 4}{\sqrt{1 + e^{-6}}} = 2\sqrt{1 + e^6}$

8. Ans. (2)

Sol. $g(f(x)) = \begin{cases} f(x), f(x) \geq 0 \\ e^{f(x)}, f(x) < 0 \end{cases}$

$g(f(x)) = \begin{cases} e^{-x}, (-\infty, 0] \\ e^{\ln x}, (0,1) \\ \ln x, [1, \infty) \end{cases}$



Graph of g(f(x))

g(f(x)) $\Rightarrow$ Many one into

9. Ans. (3)

Sol. $f(x) = \dfrac{\sqrt{x^2 - 25}}{(4 - x^2)} + \log_{10}(x^2 + 2x - 15)$

Domain : $x^2 - 25 \geq 0 \Rightarrow x \in (-\infty, -5] \cup [5, \infty)$

$4 - x^2 \neq 0 \Rightarrow x \neq \{-2, 2\}$

$x^2 + 2x - 15 > 0 \Rightarrow (x + 5)(x - 3) > 0$

$\Rightarrow x \in (-\infty, -5) \cup (3, \infty)$

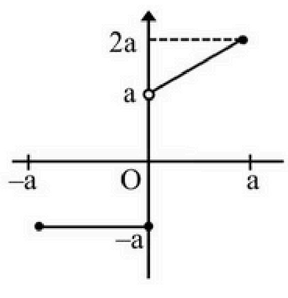
$\therefore x \in (-\infty, -5) \cup [5, \infty)$

$\alpha = -5; \beta = 5$

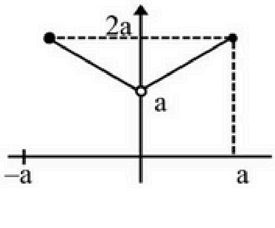
$\therefore \alpha^2 + \beta^3 = 150$

10. Ans. (1)

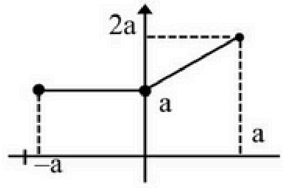
Sol. y = f(x)



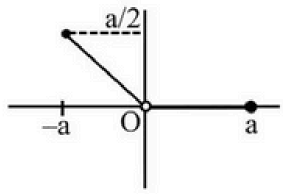
y = f|x|



y = |f(x)|



$g(x) = \dfrac{f(|x|) - |f(x)|}{2}$



11. Ans. (24)

Sol. 2x + 3y = 23

x = 1   y = 7

x = 4   y = 5

x = 7   y = 3

x = 10   y = 1

A	B
(1, 7)	1
(4, 5)	4
(7, 3)	7
(10, 1)	10

The number of one-one functions from A to B is equal to 4!

12. Ans. (1010)

Sol. f (m + n) = f(m) + f(n)

$\Rightarrow f(x) = kx$

$\Rightarrow f(1) = 1$

$\Rightarrow k = 1$

$f(x) = x$

Now

$\sum_{k=1}^{2022} f(\lambda + k) \leq (2022)^2$

$\Rightarrow \sum_{k=1}^{2022} (\lambda + k) \leq (2022)^2$

$\Rightarrow 2022\lambda + \dfrac{2022 \times 2023}{2} \leq (2022)^2$

$\Rightarrow \lambda \leq 2022 - \dfrac{2023}{2}$

$\Rightarrow \lambda \leq 1010.5$

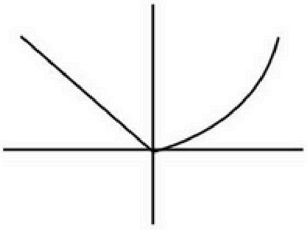
$\therefore$ largest natural no. λ is 1010.

13. Ans. (1)

Sol. $f(g(x)) = |g(x) - 1|$

$$fog\begin{bmatrix} |e^x - 1| & x \geq 0 \\ |x+1-1| & x \leq 0 \end{bmatrix}$$

$$fog\begin{bmatrix} e^x - 1 & x \geq 0 \\ -x & x \leq 0 \end{bmatrix}$$



14. Ans. (1024)

Sol. $f(f(x)) = \dfrac{2f(x)}{\sqrt{1+9f^2(x)}} = \dfrac{4x}{\sqrt{1+9x^2+9.2^2x^2}}$

$$f(f(f(x))) = \frac{2^3x/\sqrt{1+9x^2}}{\sqrt{1+9(1+2^2)\frac{2^2x^2}{1+9x^2}}} = \frac{2^3x}{\sqrt{1+9x^2(1+2^2+2^4)}}$$

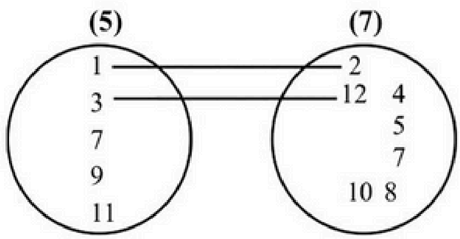
$\therefore$ By observation

$$\alpha = 1+2^2+2^4+....+2^{18} = 1\left(\frac{(2^2)^{10}-1}{2^2-1}\right) = \frac{2^{20}-1}{3}$$

$$3\alpha + 1 = 2^{20} \rightarrow \sqrt{3\alpha+1} = 2^{10} = \boxed{1024}$$

15. Ans. (4)

Sol.



A = {1, 3, 7, 9, 11}

B = {2, 4, 5, 7, 8, 10, 12}

f(1) + f(3) = 14

(i) 2 + 12

(ii) 4 + 10

2 × (2 × 5 × 4 × 3) = 240

16. Ans. (18)

Sol. $|2a - 1| = 3[a] + 2\{a\}$

$|2a - 1| = [a] + 2a$

Case-1 : $a > \dfrac{1}{2}$

$2a - 1 = [a] + 2a$

$[a] = -1 \quad \therefore a \in [-1, 0)$ Reject

Case-2 : $a < \dfrac{1}{2}$

$-2a + 1 = [a] + 2a$

$a = I + f$

$-2(I + f) + 1 = I + 2I + 2f$

$I = 0,\ f = \dfrac{1}{4} \quad \therefore\ a = \dfrac{1}{4}$

Hence $a = \dfrac{1}{4}$

$$72\sum_{a\in S} a = 72 \times \frac{1}{4} = 18$$

17. Ans. (4)

Sol. $\sin 5x \in [-1,1]$

$-\sin 5x \in [-1, 1]$

$7 - \sin 5x \in [6, 8]$

$$\frac{1}{7 - \sin 5x} \in \left[\frac{1}{8}, \frac{1}{6}\right]$$

18. Ans. NTA (3)

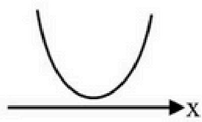
Allen (Bonus)

Sol. $f(x) = \dfrac{(x+5)(x-3)}{x^2 - 4x + 9}$

Let $g(x) = x^2 - 4x + 9$

$D < 0$

$g(x) > 0$ for $x \in R$



$$\therefore \begin{bmatrix} f(-5) = 0 \\ f(3) = 0 \end{bmatrix}$$

So, f(x) is many-one.

again,

$yx^2 - 4xy + 9y = x^2 + 2x - 15$

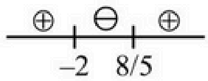
$x^2 (y - 1) - 2x(2y + 1) + (9y + 15) = 0$

for $\forall\, x \in R \Rightarrow D \geq 0$

$D = 4(2y + 1)^2 - 4(y - 1)(9y + 15) \geq 0$

$5y^2 + 2y + 16 \leq 0$

$(5y - 8)(y + 2) \leq 0$



$\boxed{y \in \left[-2, \frac{8}{5}\right]}$ range

Note : If function is defined from f : R → R then only correct answer is option (3)

⇒ Bonus

19. Ans. (2)

Sol. $N = 2310 = 231 \times 10$

$= 3 \times 11 \times 7 \times 2 \times 5$

$A = \{2, 3, 5, 7, 11\}$

$$f(x) = \left[\log_2\left(x^2 + \left[\frac{x^3}{5}\right]\right)\right]$$

$f(2) = [\log_2(5)] = 2$

$f(3) = [\log_2(14)] = 3$

$f(5) = [\log_2(25 + 25)] = 5$

$f(7) = [\log_2(117)] = 6$

$f(11) = [\log_2 387] = 8$

Range of f : B = {2, 3, 5, 6, 8}

No. of one-one functions = 5! = 120