

GRAVITATION

10. If R is the radius of the earth and the acceleration due to gravity on the surface of earth is $g = \pi^2 \text{ m/s}^2$, then the length of the second's pendulum at a height $h = 2R$ from the surface of earth will be,:

(1) $\frac{2}{9}m$ (2) $\frac{1}{9}m$

(3) $\frac{4}{9}m$ (4) $\frac{8}{9}m$

11. A light planet is revolving around a massive star in a circular orbit of radius R with a period of revolution T . If the force of attraction between planet and star is proportional to $R^{-3/2}$ then choose the correct option :

(1) $T^2 \propto R^{5/2}$ (2) $T^2 \propto R^{7/2}$

(3) $T^2 \propto R^{3/2}$ (4) $T^2 \propto R^3$

12. A metal wire of uniform mass density having length L and mass M is bent to form a semicircular arc and a particle of mass m is placed at the centre of the arc. The gravitational force on the particle by the wire is:

(1) $\frac{GMm\pi}{2L^2}$ (2) 0

(3) $\frac{GmM\pi^2}{L^2}$ (4) $\frac{2GmM\pi}{L^2}$

13. Correct formula for height of a satellite from earth's surface is :

(1) $\left(\frac{T^2 R^2 g}{4\pi}\right)^{1/2} - R$ (2) $\left(\frac{T^2 R^2 g}{4\pi^2}\right)^{1/3} - R$

(3) $\left(\frac{T^2 R^2}{4\pi^2 g}\right)^{1/3} - R$ (4) $\left(\frac{T^2 R^2}{4\pi^2}\right)^{-1/3} + R$

14. A 90 kg body placed at $2R$ distance from surface of earth experiences gravitational pull of :

(R = Radius of earth, $g = 10 \text{ ms}^{-2}$)

(1) 300 N (2) 225 N

(3) 120 N (4) 100 N

15. In hydrogen like system the ratio of coulombian force and gravitational force between an electron and a proton is in the order of:

(1) 10^{39} (2) 10^{19}

(3) 10^{29} (4) 10^{36}

16. Match list-I with list-II:

	List-I		List-II
(A)	Kinetic energy of planet	(I)	$\frac{GMm}{a}$
(B)	Gravitation Potential energy of Sun-planet system.	(II)	$\frac{GMm}{2a}$
(C)	Total mechanical energy of planet	(III)	$\frac{Gm}{r}$
(D)	Escape energy at the surface of planet for unit mass object	(IV)	$\frac{GMm}{2a}$

(Where a = radius of planet orbit, r = radius of planet, M = mass of Sun, m = mass of planet)

Choose the correct answer from the options given below:

(1) (A) – II, (B) – I, (C) – IV, (D) – III

(2) (A) – III, (B) – IV, (C) – I, (D) – II

(3) (A) – I, (B) – IV, (C) – II, (D) – III

(4) (A) – I, (B) – II, (C) – III, (D) – IV

17. A simple pendulum doing small oscillations at a place R height above earth surface has time period of $T_1 = 4$ s. T_2 would be it's time period if it is brought to a point which is at a height 2R from earth surface. Choose the correct relation [R = radius of Earth]:
- (1) $T_1 = T_2$ (2) $2T_1 = 3T_2$
 (3) $3T_1 = 2T_2$ (4) $2T_1 = T_2$
18. A satellite revolving around a planet in stationary orbit has time period 6 hours. The mass of planet is one-fourth the mass of earth. The radius orbit of planet is : (Given = Radius of geo-stationary orbit for earth is 4.2×10^4 km)
- (1) 1.4×10^4 km
 (2) 8.4×10^4 km
 (3) 1.68×10^5 km
 (4) 1.05×10^4 km
19. To project a body of mass m from earth's surface to infinity, the required kinetic energy is (assume, the radius of earth is R_E , g = acceleration due to gravity on the surface of earth) :
- (1) $2mgR_E$ (2) mgR_E
 (3) $\frac{1}{2}mgR_E$ (4) $4mgR_E$
20. Assuming the earth to be a sphere of uniform mass density, a body weighed 300 N on the surface of earth. How much it would weigh at R/4 depth under surface of earth ?
- (1) 75 N (2) 375 N
 (3) 300 N (4) 225 N
21. Two planets A and B having masses m_1 and m_2 move around the sun in circular orbits of r_1 and r_2 radii respectively. If angular momentum of A is L and that of B is 3L, the ratio of time period $\left(\frac{T_A}{T_B}\right)$ is:
- (1) $\left(\frac{r_2}{r_1}\right)^{\frac{3}{2}}$ (2) $\left(\frac{r_1}{r_2}\right)^3$
 (3) $\frac{1}{27}\left(\frac{m_2}{m_1}\right)^3$ (4) $27\left(\frac{m_1}{m_2}\right)^3$
22. Two satellite A and B go round a planet in circular orbits having radii 4 R and R respectively. If the speed of A is 3v, the speed of B will be :
- (1) $\frac{4}{3}v$ (2) 3v
 (3) 6v (4) 12v
23. An astronaut takes a ball of mass m from earth to space. He throws the ball into a circular orbit about earth at an altitude of 318.5 km. From earth's surface to the orbit, the change in total mechanical energy of the ball is $x \frac{GM_E m}{21R_e}$. The value of x is (take $R_e = 6370$ km):
- (1) 11 (2) 9
 (3) 12 (4) 10
24. A satellite of 10^3 kg mass is revolving in circular orbit of radius 2R. If $\frac{10^4 R}{6}$ J energy is supplied to the satellite, it would revolve in a new circular orbit of radius :
- (use $g = 10 \text{ m/s}^2$, R = radius of earth)
- (1) 2.5 R (2) 3 R
 (3) 4 R (4) 6 R

GRAVITATION
SOLUTIONS
1. Ans. (4)

Sol. $g = \frac{GM}{R^2} \Rightarrow g \propto \frac{1}{R^2}$

$$\frac{g_2}{g_1} = \frac{R_1^2}{R_2^2}; g_2 = 4g_1 \left(R_2 = \frac{R_1}{2} \right)$$

2. Ans. (2)

Sol. $\omega = \frac{2\pi}{T} \Rightarrow \omega \propto \frac{1}{T}$

$T_{\text{moon}} = 27 \text{ days}$

$T_{\text{earth}} = 365 \text{ days } 4 \text{ hour}$

$\Rightarrow \omega_{\text{moon}} > \omega_{\text{earth}}$

3. Ans. (4)

Sol. $g_p = \frac{gR^2}{(R+h)^2}$

$g_q = g \left(1 - \frac{h}{R} \right)$

$g_p = g_q$

$$\frac{g}{\left(1 + \frac{h}{R} \right)^2} = g \left(1 - \frac{h}{R} \right)$$

$$\left(1 - \frac{h^2}{R^2} \right) \left(1 + \frac{h}{R} \right) = 1$$

Take $\frac{h}{R} = x$

So

$x^3 - x + x^2 = 0$

$x = \frac{\sqrt{5} - 1}{2}$

$h = \frac{R}{2} (\sqrt{5} - 1)$

4. Ans. (1)

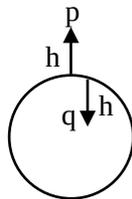
Sol. $T^2 \propto r^3$

$\frac{T_1^2}{r_1^3} = \frac{T_2^2}{r_2^3}$

$$\frac{(200)^2}{r^3} = \frac{T_2^2}{\left(\frac{r}{4} \right)^3}$$

$\frac{200 \cdot 200}{4 \cdot 4 \cdot 4} = T_2^2$

$T_2 = \frac{200}{4 \cdot 2} \quad T_2 = 25 \text{ days}$


5. Ans. (1)

Sol. $-\frac{GM_E}{R_E + h} = -5.12 \cdot 10^{-7} \dots (i)$

$$\frac{GM_E}{(R_E + h)^2} = 6.4 \dots (ii)$$

By (i) and (ii)

$\Rightarrow h = 16 \cdot 10^5 \text{ m } = 1600 \text{ km}$

6. Ans. (4)

Sol. $R_p = \frac{R_E}{3}, M_p = \frac{M_E}{6}$

$V_e = \sqrt{\frac{2GM_e}{R_e}} \dots (i)$

$V_p = \sqrt{\frac{2GM_p}{R_p}} \dots (ii)$

$\frac{V_e}{V_p} = \sqrt{2}$

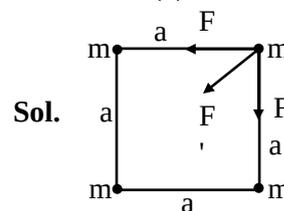
$V_p = \frac{V_e}{\sqrt{2}} = \frac{11.2}{\sqrt{2}} = 7.9 \text{ km/sec}$

7. Ans. (8)
Sol. Acceleration due to gravity $g' = \frac{g}{4}$

$T = 2\pi \sqrt{\frac{4R}{g}}$

$T = 2\pi \sqrt{\frac{4 \cdot 4}{g}}$

$T = 2\pi \frac{4}{\pi} = 8 \text{ s}$

8. Ans. (2)

Sol.

$F_{\text{net}} = \sqrt{2}F + F'$

$F = \frac{Gm^2}{a^2} \text{ and } F' = \frac{Gm^2}{(\sqrt{2}a)^2}$

$$F_{\text{net}} = \sqrt{2} \frac{Gm^2}{a^2} + \frac{Gm^2}{2a^2}$$

$$\left(\frac{2\sqrt{2}+1}{32} \right) \frac{Gm^2}{L^2} = \frac{Gm^2}{a^2} \left(\frac{2\sqrt{2}+1}{2} \right)$$

$$a = 4L$$

9. **Ans. (1)**

Sol. $V_{\text{escape}} = \sqrt{\frac{2GM}{R}}$

$$V_{\text{planet}} = \sqrt{\frac{2GM}{R}} = V$$

$$V_{\text{Moon}} = \sqrt{\frac{2GM \cdot 16}{144R}} = \frac{1}{3} \sqrt{\frac{2GM}{R}}$$

$$V_{\text{Moon}} = \frac{V_{\text{planet}}}{3} = \frac{V}{3}$$

10. **Ans. (2)**

Sol. $g' = \frac{GM_e}{(3R)^2} = \frac{1}{9}g$

$$T = 2\pi \sqrt{\frac{l}{g'}}$$

Since the time period of second pendulum is 2 sec.

$$T = 2 \text{ sec}$$

$$2 = 2\pi \sqrt{\frac{l}{g}}$$

$$l = \frac{1}{9}m$$

11. **Ans. (1)**

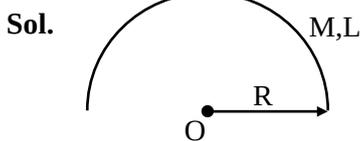
Sol. $F = \frac{GMm}{R^{3/2}} = m\omega^2 R$

$$\omega^2 \propto \frac{1}{R^{5/2}}$$

$$\therefore T = \frac{2\pi}{\omega} \text{ so}$$

$$T^2 \propto R^{5/2}$$

12. **Ans. (4)**



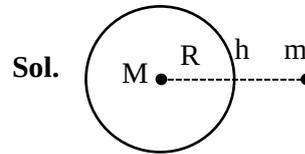
$$\text{We have } R = \frac{L}{\pi}$$

$$g_0 = \frac{2G \frac{M}{L}}{R} = \frac{2GM\pi}{L^2}$$

$$\therefore F_m = mg_0 = \frac{2GM\pi m}{L^2}$$

Hence option (4)

13. **Ans. (2)**



$$\Rightarrow \frac{GMm}{(R+h)^2} = \frac{mv^2}{(R+h)}$$

$$\Rightarrow \frac{GM}{(R+h)} = v^2 \dots (1)$$

$$\Rightarrow v = (R+h)\omega$$

$$\Rightarrow v = (R+h) \frac{2\pi}{T} \dots (2)$$

$$\Rightarrow \frac{GM}{R^2} = g$$

$$\Rightarrow GM = gR^2 \dots (3)$$

Put value from (2) & (3) in eq. (1)

$$\Rightarrow \frac{gR^2}{(R+h)} = (R+h)^2 \left(\frac{2\pi}{T} \right)^2$$

$$\Rightarrow \frac{T^2 R^2 g}{(2\pi)^2} = (R+h)^3$$

$$\Rightarrow \left[\frac{T^2 R^2 g}{(2\pi)^2} \right]^{1/3} - R = h$$

14. **Ans. (4)**

Sol. Value of $g = g_s \left(1 + \frac{h}{R} \right)^{-2}$

$$= g_s (1+2)^{-2} = \frac{g_s}{9}$$

Here g_s = gravitational acceleration at surface

$$\text{Force} = mg = 90 \times \frac{g_s}{9} = 100 \text{ N}$$

15. **Ans. (1)**

Sol. $F_e = \frac{kQ_1 Q_2}{r^2} = \frac{9 \cdot 10^9 \cdot 1.6 \cdot 10^{-19} \cdot 1.6 \cdot 10^{-19}}{r^2}$

$$F_g = \frac{Gm_1 m_2}{r^2} = \frac{6.67 \cdot 10^{-11} \cdot 9.1 \cdot 10^{-31} \cdot 1.6 \cdot 10^{-27}}{r^2}$$

$$\frac{F_e}{F_g} \approx 0.23 \cdot 10^{40} \approx 2.3 \cdot 10^{39}$$

Option (1)

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16. Ans. (1)

Sol. $KE = \frac{1}{2}mv^2 = \frac{GMm}{2a}$

$PE = -2KE$

$TE = -KE$

17. Ans. (3)

Sol. $T_1 = 2\pi\sqrt{\frac{\square}{GM}}(2R)^2$

$T_2 = 2\pi\sqrt{\frac{\square}{GM}}(3R)^2 \therefore \frac{T_1}{T_2} = \frac{2}{3}$

18. Ans. (4)

Sol. $T = \frac{2\pi}{\sqrt{GM}}$

$\frac{T_1}{T_2} = \left(\frac{r_1}{r_2}\right)^{3/2} \left(\frac{M_2}{M_1}\right)^{1/2}$

$\frac{6}{24} = \frac{(r_1)^{3/2}}{(4.2 \cdot 10^4)^{3/2}} \left(\frac{M}{M/4}\right)^{1/2}$

$r_1 = 1.05 \times 10^4 \text{ km}$

19. Ans. (2)

Sol. $\frac{1}{2}mv_e^2 = \frac{GMm}{R_E}$

$g = \frac{GM}{R_E^2}$

$K = mgR_E$

20. Ans. (4)

Sol. At surface: $mg = 300 \text{ N}$

$m = \frac{300}{g_s}$

At Depth $\frac{R}{4}$: $g_d = g_s \left[1 - \frac{d}{R}\right]$

$g_d = g_s \left[1 - \frac{R}{4R}\right]; g_d = \frac{3g_s}{4}$

weight at depth = $m \times g_d$

$= m \cdot \frac{3g_s}{4} = \frac{3}{4} \cdot 300 = 225 \text{ N}$

21. Ans. (3)

Sol. $\frac{\pi r_1^2}{T_A} = \frac{L}{2m_1} \dots\dots (1)$

$\frac{\pi r_2^2}{T_B} = \frac{3L}{2m_2} \dots\dots (2)$

$\Rightarrow \frac{T_A}{T_B} = 3 \cdot \frac{m_1}{m_2} \cdot \left(\frac{r_1}{r_2}\right)^2$

$\left(\frac{T_A}{T_B}\right)^2 = \left(\frac{r_1}{r_2}\right)^3 \Rightarrow \left(\frac{r_1}{r_2}\right)^2 = \left(\frac{T_A}{T_B}\right)^{\frac{4}{3}}$

$\Rightarrow \frac{1}{27} \cdot \left(\frac{m_2}{m_1}\right)^3 = \left(\frac{T_A}{T_B}\right)$

22. Ans. (3)

Sol. $v = \sqrt{\frac{GM}{R}}$

$\frac{v_A}{v_B} = \sqrt{\frac{R_B}{R_A}} = \sqrt{\frac{R}{4R}} = \frac{1}{2}$

$v_B = 2v_A = 6v$

23. Ans. (1)

Sol. $h = 318.5 \approx \left(\frac{R_e}{20}\right)$

$TE_i = \frac{-GM_e m}{R_e}$

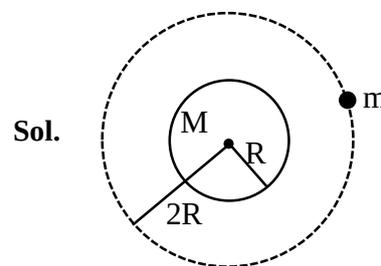
$TE_f = \frac{-GM_e m}{2(R_e + h)} = \frac{-GM_e m}{2\left(R_e + \frac{R_e}{20}\right)}$

$\Rightarrow TE_f = \frac{-10GM_e m}{21R_e}$

Change in total mechanical energy

$= TE_f - TE_i$

$= \frac{GM_e m}{R_e} \left[1 - \frac{10}{21}\right] = \frac{11GM_e m}{21R_e}$

24. Ans. (4)


Total energy = $\frac{-GMm}{2(2R)}$

 if energy = $\frac{10^4 R}{6}$ is added then

$\frac{-GMm}{4R} + \frac{10^4 R}{6} = \frac{-GMm}{2r}$

 where r is new radius of revolving and $g = \frac{GM}{R^2}$

$\frac{mgR}{4} + \frac{10^4 R}{6} = \frac{mgR^2}{2r} \quad (m = 10^3 \text{ kg})$

$\frac{10^3 \cdot 10 \cdot R}{4} + \frac{10^4 R}{6} = \frac{10^3 \cdot 10 \cdot R^2}{2r}$

$\frac{1}{4} + \frac{1}{6} = \frac{R}{2r}$

$r = 6R$