

METHOD OF DIFFERENTIATION

1. Let $f(x) = x^3 + x^2 f'(1) + x f''(2) + f'''(3)$, $x \in \mathbb{R}$.
Then $f'(10)$ is equal to _____.

2. Suppose

$$f(x) = \frac{(2^x + 2^{-x}) \tan x \sqrt{\tan^{-1}(x^2 - x + 1)}}{(7x^2 + 3x + 1)^3}$$

Then the value of $f'(0)$ is equal to

(1) π (2) 0

(3) $\sqrt{\pi}$ (4) $\frac{\pi}{2}$

3. Let $f(x) = 2^x - x^2$, $x \in \mathbb{R}$. If m and n are respectively the number of points at which the curves $y = f(x)$ and $y = f'(x)$ intersects the x -axis, then the value of $m + n$ is

4. Let $y = \log_e \left(\frac{1-x^2}{1+x^2} \right)$, $-1 < x < 1$. Then at $x = \frac{1}{2}$,

the value of $225(y' - y'')$ is equal to

(1) 732 (2) 746

(3) 742 (4) 736

5. If $f(x) = \begin{vmatrix} 2 \cos^4 x & 2 \sin^4 x & 3 + \sin^2 2x \\ 3 + 2 \cos^4 x & 2 \sin^4 x & \sin^2 2x \\ 2 \cos^4 x & 3 + 2 \sin^4 x & \sin^2 2x \end{vmatrix}$

then $\frac{1}{5} f'(0)$ is equal to _____

(1) 0 (2) 1

(3) 2 (4) 6

6. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a function satisfying

$$f\left(\frac{x}{y}\right) = \frac{f(x)}{f(y)} \text{ for all } x, y, f(y) \neq 0.$$

If $f(1) = 2024$, then

(1) $xf'(x) - 2024 f(x) = 0$

(2) $xf'(x) + 2024 f(x) = 0$

(3) $xf'(x) + f(x) = 2024$

(4) $xf'(x) - 2023 f(x) = 0$

7. If $f(x) = \begin{vmatrix} x^3 & 2x^2 + 1 & 1 + 3x \\ 3x^2 + 2 & 2x & x^3 + 6 \\ x^3 - x & 4 & x^2 - 2 \end{vmatrix}$

for all $x \in \mathbb{R}$, then $2f(0) + f'(0)$ is equal to

(1) 48 (2) 24

(3) 42 (4) 18

8. If

$$y = \frac{(\sqrt{x} + 1)(x^2 - \sqrt{x})}{x\sqrt{x} + x + \sqrt{x}} + \frac{1}{15}(3\cos^2 x - 5)\cos^3 x,$$

then $96y' \left(\frac{\pi}{6} \right)$ is equal to :

9. Let $f(x) = ax^3 + bx^2 + cx + 41$ be such that

$f(1) = 40$, $f'(1) = 2$ and $f''(1) = 4$.

Then $a^2 + b^2 + c^2$ is equal to :

(1) 62 (2) 73

(3) 54 (4) 51

10. Let $f(x) = x^5 + 2e^{x/4}$ for all $x \in \mathbb{R}$. Consider a function $g(x)$ such that $(g \circ f)(x) = x$ for all $x \in \mathbb{R}$.

Then the value of $8g'(2)$ is :

(1) 16 (2) 4

(3) 8 (4) 2

11. If $y(\theta) = \frac{2\cos\theta + \cos 2\theta}{\cos 3\theta + 4\cos 2\theta + 5\cos\theta + 2}$,

then at $\theta = \frac{\pi}{2}$, $y'' + y' + y$ is equal to :

(1) $\frac{3}{2}$ (2) 1

(3) $\frac{1}{2}$ (4) 2

SOLUTIONS

1. Ans. (202)

Sol. $f(x) = x^3 + x^2 \cdot f'(1) + x \cdot f''(2) + f'''(3)$

$$f'(x) = 3x^2 + 2xf'(1) + f''(2)$$

$$f''(x) = 6x + 2f'(1)$$

$$f'''(x) = 6$$

$$f'(1) = -5, f''(2) = 2, f'''(3) = 6$$

$$f(x) = x^3 + x^2 \cdot (-5) + x \cdot (2) + 6$$

$$f'(x) = 3x^2 - 10x + 2$$

$$f'(10) = 300 - 100 + 2 = 202$$

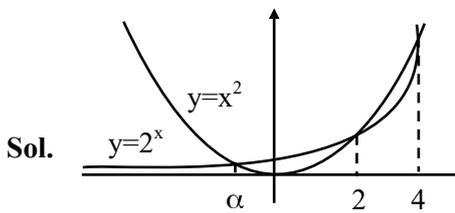
2. Ans. (3)

Sol. $f'(0) = \lim_{h \rightarrow 0} \frac{f(h) - f(0)}{h}$

$$= \lim_{h \rightarrow 0} \frac{(2^h + 2^{-h}) \tan h \sqrt{\tan^{-1}(h^2 - h + 1)} - 0}{(7h^2 + 3h + 1)^3 h}$$

$$= \sqrt{\pi}$$

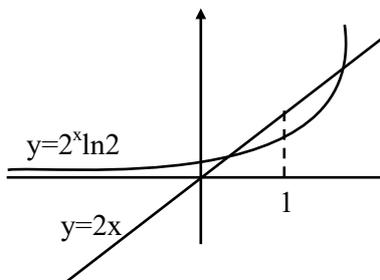
3. Ans. (5)



$$\therefore m = 3$$

$$f'(x) = 2^x \ln 2 - 2x = 0$$

$$2^x \ln 2 = 2x$$



$$\therefore n = 2$$

$$\Rightarrow m + n = 5$$

4. Ans. (4)

Sol. $y = \log_e \left(\frac{1-x^2}{1+x^2} \right)$

$$\frac{dy}{dx} = y' = \frac{-4x}{1-x^4}$$

Again,

$$\frac{d^2y}{dx^2} = y'' = \frac{-4(1+3x^4)}{(1-x^4)^2}$$

Again

$$y' - y'' = \frac{-4x}{1-x^4} + \frac{4(1+3x^4)}{(1-x^4)^2}$$

$$\text{at } x = \frac{1}{2},$$

$$y' - y'' = \frac{736}{225}$$

$$\text{Thus } 225(y' - y'') = 225 \times \frac{736}{225} = 736$$

5. Ans. (1)

Sol.

$$\begin{vmatrix} 2 \cos^4 x & 2 \sin^4 x & 3 + \sin^2 2x \\ 3 + 2 \cos^4 x & 2 \sin^4 x & \sin^2 2x \\ 2 \cos^4 x & 3 + 2 \sin^2 4x & \sin^2 2x \end{vmatrix}$$

$$R_2 \rightarrow R_2 - R_1, R_3 \rightarrow R_3 - R_1$$

$$\begin{vmatrix} 2 \cos^4 x & 2 \sin^4 x & 3 + \sin^2 2x \\ 3 & 0 & -3 \\ 0 & 3 & -3 \end{vmatrix}$$

$$f(x) = 45$$

$$f'(x) = 0$$

6. Ans. (1)

Sol. $f\left(\frac{x}{y}\right) = \frac{f(x)}{f(y)}$ $f(1) = 2024$
 $f(1) = 1$

Partially differentiating w. r. t. x

$$f'\left(\frac{x}{y}\right) \cdot \frac{1}{y} = \frac{1}{f(y)} f'(x)$$

$$y \rightarrow x$$

$$f(1) \cdot \frac{1}{x} = \frac{f(x)}{f(x)}$$

$$2024f(x) = xf'(x) \Rightarrow xf'(x) - 2024f(x) = 0$$

METHOD OF DIFFERENTIATION

7. Ans. (3)

Sol. $f(0) = \begin{vmatrix} 0 & 1 & 1 \\ 2 & 0 & 6 \\ 0 & 4 & -2 \end{vmatrix} = 12$

$$f'(x) = \begin{vmatrix} 3x^2 & 4x & 3 \\ 3x^2 + 2 & 2x & x^3 + 6 \\ x^3 - x & 4 & x^2 - 2 \end{vmatrix} +$$

$$\begin{vmatrix} x^3 & 2x^2 + 1 & 1 + 3x \\ 6x & 2 & 3x^2 \\ x^3 - x & 4 & x^2 - 2 \end{vmatrix} +$$

$$\begin{vmatrix} x^3 & 2x^2 + 1 & 1 + 3x \\ 3x^2 + 2 & 2x & x^3 + 6 \\ 3x^2 - 1 & 0 & 2x \end{vmatrix}$$

$$\therefore f'(0) = \begin{vmatrix} 0 & 0 & 3 \\ 2 & 0 & 6 \\ 0 & 4 & -2 \end{vmatrix} + \begin{vmatrix} 0 & 1 & 1 \\ 0 & 2 & 0 \\ 0 & 4 & -2 \end{vmatrix} + \begin{vmatrix} 0 & 1 & 1 \\ 2 & 0 & 6 \\ -1 & 0 & 0 \end{vmatrix}$$

$$= 24 - 6 = 18$$

$$\therefore 2f(0) + f'(0) = 42$$

8. Ans. (105)

Sol. $y = \frac{(\sqrt{x} + 1)(x^2 - \sqrt{x})}{x\sqrt{x} + x + \sqrt{x}} + \frac{1}{15}(3\cos^2 x - 5)\cos^3 x$

$$y = \frac{(\sqrt{x} + 1)(\sqrt{x})((\sqrt{x})^3 - 1)}{(\sqrt{x})((\sqrt{x})^2 + (\sqrt{x}) + 1)} + \frac{1}{5}\cos^5 x - \frac{1}{3}\cos^3 x$$

$$y = (\sqrt{x} + 1)(\sqrt{x} - 1) + \frac{1}{5}\cos^5 x - \frac{1}{3}\cos^3 x$$

$$y' = 1 - \cos^4 x \cdot (\sin x) + \cos^2 x (\sin x)$$

$$y' \left(\frac{\pi}{6} \right) = 1 - \frac{9}{16} \cdot \frac{1}{2} + \frac{3}{4} \cdot \frac{1}{2}$$

$$= \frac{32 - 9 + 12}{32} = \frac{35}{32}$$

$$= 96y' \left(\frac{\pi}{6} \right) = 105$$

9. Ans. (4)

Sol. $f(x) = ax^3 + bx^2 + cx + 41$

$$f'(x) = 3ax^2 + 2bx + cx$$

$$\Rightarrow f'(1) = 3a + 2b + c = 2 \dots\dots\dots(1)$$

$$f''(x) = 6ax + 2b$$

$$\Rightarrow f''(1) = 6a + 2b = 4$$

$$3a + b = 2 \dots\dots\dots(2)$$

$$(1) - (2)$$

$$b + c = 0 \dots\dots\dots(3)$$

$$f(1) = 40$$

$$a + b + c + 41 = 40$$

use (3)

$$a + 41 = 40$$

by (2)

$$-3 + b = 2 \Rightarrow b = 5 \text{ \& } c = -5$$

$$a^2 + b^2 + c^2 = 1 + 25 + 25 = 51$$

10. Ans. (1)

Sol. $f(x) = 2$

when $x = 0$

$$\therefore g'(f(x)) f'(x) = 1$$

$$g'(2) = \frac{1}{f'(0)}$$

$$\therefore f'(x) = 5x^4 + \frac{2}{4}e^{x/4}$$

$$g'(2) = 2$$

Ans = 16

Option (1)

11. Ans. (4)

$$\text{Sol. } y = \frac{2\cos\theta + 2\cos^2\theta - 1}{4\cos^3\theta - 3\cos\theta + 8\cos^2\theta - 4 + 5\cos\theta + 2}$$

$$y = \frac{(2\cos^2\theta + 2\cos\theta - 1)}{(2\cos^2\theta + 2\cos\theta - 1)(2\cos\theta + 2)}$$

$$y = \frac{1}{2} \left(\frac{1}{1 + \cos\theta} \right)$$

$$\Rightarrow \theta = \frac{\pi}{2} \quad y = \frac{1}{2}$$

$$y' = \frac{1}{2} \left(\frac{-1}{(1 + \cos\theta)^2} \cdot (-\sin\theta) \right)$$

$$\Rightarrow \theta = \frac{\pi}{2} \quad y = \frac{1}{2}$$

$$y'' = \frac{1}{2} \left[\frac{\cos\theta(1 + \cos\theta)^2 - \sin\theta(2)(1 + \cos\theta)(-\sin\theta)}{(1 + \cos\theta)^4} \right]$$

$$\Rightarrow \theta = \frac{\pi}{2} \quad y = 1$$

12. Ans. (4)

$$\text{Sol. } f(x) = x^5 + 2x^3 + 3x + 1$$

$$f'(x) = 5x^4 + 6x^2 + 3$$

$$f'(1) = 5 + 6 + 3 = 14$$

$$g(f(x)) = x$$

$$g'(f(x))f'(x) = 1$$

$$\text{for } f(x) = 7$$

$$\Rightarrow x^5 + 2x^3 + 3x + 1 = 7$$

$$\Rightarrow x = 1$$

$$g'(7)f'(1) = 1 \Rightarrow g'(7) = \frac{1}{f'(1)} = \frac{1}{14}$$

$$x = 1, f(x) = 7 \Rightarrow g(7) = 1$$

$$\frac{g(7)}{g'(7)} = \frac{1}{1/14} = 14$$

13. Ans. (4)

$$\text{Sol. } g(x) = h(e^x) \cdot e^{h(x)}$$

$$g'(x) = h(e^x) \cdot e^{h(x)} \cdot h'(x) + e^{h(x)} h'(e^x) \cdot e^x$$

$$g'(0) = h(1)e^{h(0)}h'(0) + e^{h(0)}h'(1)$$

$$= 2 + 2 = 4$$

14. Ans. (2)

$$\text{Sol. } f'(x) = 3x^2 \sin\left(\frac{1}{x}\right) - x \cos\left(\frac{1}{x}\right)$$

$$f''(x) = 6x \sin\left(\frac{1}{x}\right) - 3\cos\left(\frac{1}{x}\right) - \cos\left(\frac{1}{x}\right) - \frac{\sin\left(\frac{1}{x}\right)}{x}$$

$$f''\left(\frac{2}{\pi}\right) = \frac{12}{\pi} - \frac{\pi}{2} = \frac{24 - \pi^2}{2\pi}$$