

Matrices and Determinants : JEE Main Past Year Solved Questions

1. Let $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$ and $B = I + \text{adj}(A) + (\text{adj } A)^2 + \dots + (\text{adj } A)^{10}$. Then, the sum of all the elements of the matrix B is :

(1) -110

(2) 22

(3) -88

(4) -124

Ans. (3)

Sol.

$$\text{Sol. Adj}(A) = \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix}$$

$$(\text{Adj } A)^2 = \begin{bmatrix} 1 & -4 \\ 0 & 1 \end{bmatrix}$$

$$(\text{Adj } A)^{10} = \begin{bmatrix} 1 & -20 \\ 0 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 & -4 \\ 0 & 1 \end{bmatrix} + \dots + \begin{bmatrix} 1 & -20 \\ 0 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 11 & -110 \\ 0 & 11 \end{bmatrix} \quad \square \text{ sum of elements of } B \\ = -88$$

2 Let A be a 2×2 symmetric matrix such that $A \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 7 \end{bmatrix}$ and the determinant of A be 1. If $A^{-1} = \alpha A + \beta I$, where I is an identity matrix of order 2×2 , then $\alpha + \beta$ equals

Ans. (5)

Sol.

$$\text{Let } A = \begin{bmatrix} a & b \\ b & d \end{bmatrix}$$

$$\begin{bmatrix} a & b \\ b & d \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 7 \end{bmatrix}, \quad ad - b^2 = 1$$

$$a + b = 3, \quad b + d = 7, \quad (3 - b)(7 - b) - b^2 = 1$$

$$21 - 10b = 1 \rightarrow b = 2, \quad a = 1, \quad d = 5$$

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 5 \end{bmatrix}, \quad A^{-1} = \begin{bmatrix} 5 & -2 \\ -2 & 1 \end{bmatrix}$$

$$A^{-1} = \alpha A + \beta I$$

$$\begin{bmatrix} 5 & -2 \\ -2 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \alpha + \beta & 2\alpha \\ 2\alpha & 5\alpha + \beta \end{bmatrix}$$

$$\alpha = -1, \quad \beta = 6 \Rightarrow \boxed{\alpha + \beta = 5}$$

3. Let $\alpha \in (0, \infty)$ and $A = \begin{bmatrix} 1 & 2 & \alpha \\ 0 & 1 & 0 \\ 1 & 0 & 2 \end{bmatrix}$ If $\det(\text{adj}(2A - A^T) \cdot \text{adj}(A - 2A^T)) = 2^8$,

then $(\det A)^2$ is equal to :

(1) 1

(2) 49

(3) 16

(4) 36

Ans. (3)

Sol.

$$\text{adj}(A - 2A^T) \cdot (2A - A^T) = 28$$

$$|(A - 2A^T)(2A - A^T)| = 24$$

$$|A - 2A^T| \cdot |2A - A^T| = \pm 16$$

$$(A - 2A^T)^T = A^T - 2A$$

$$|A - 2A^T| = |A^T - 2A| \Rightarrow |A - 2A^T|^2 = 16 \Rightarrow |A - 2A^T| = \pm 4$$

$$\begin{bmatrix} 1 & 2 & \alpha \\ 0 & 1 & 0 \\ 1 & 0 & 2 \end{bmatrix} \Rightarrow A^T = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 1 & 0 \\ \alpha & 0 & 2 \end{bmatrix}$$

$$A - 2A^T = \begin{bmatrix} 1 & 2 & \alpha \\ 0 & 1 & 0 \\ 1 & 0 & 2 \end{bmatrix} - 2 \begin{bmatrix} 1 & 0 & 1 \\ 2 & 1 & 0 \\ \alpha & 0 & 2 \end{bmatrix} = \begin{bmatrix} -1 & 2 & \alpha - 2 \\ -3 & -1 & 0 \\ 1 - 2\alpha & 0 & -2 \end{bmatrix}$$

$$1 + 3\alpha = 4 \Rightarrow 3\alpha = 3 \Rightarrow \alpha = 1$$

$$\alpha = 1$$

$$|A| = \begin{vmatrix} 1 & 2 & 1 \\ 0 & 1 & -1 \\ -1 & 3 & -4 \end{vmatrix} = -1 - 3 = -4$$

$$|A|^2 = 16$$

4. Let $\alpha \neq 0$ and $\beta \neq 0$. If $A = \begin{bmatrix} \beta & \alpha & 3\alpha \\ \alpha & \alpha & \beta \\ -\beta & \alpha & 2\alpha \end{bmatrix}$, $B = \begin{bmatrix} 3\alpha & -9 & 3\alpha \\ -\alpha & 7 & -2\alpha \\ -2\alpha & 5 & -2\beta \end{bmatrix}$, $\alpha\beta \neq 0$ is the matrix of

cofactors of the elements of A, then $\det(AB)$ is equal to :

(1) 343

(2) 125

(3) 64

(4) 216

Ans. (4)

Sol. Equating co-factor fo A_{21}

$$A_{21} = (2\alpha^2 - 3\alpha) = \alpha$$

$$\Rightarrow \alpha = 0, 2 \quad (\text{accept})$$

$$2\alpha^2 - \alpha\beta = 3\alpha$$

$$\Rightarrow \alpha = 2, \beta = 1$$

$$|AB| = |A \cdot \text{cof}(A)| = |A|^3$$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 2 & 1 \\ -1 & 2 & 4 \end{bmatrix}$$

$$\Rightarrow |A| = 6 - 2(9) + 3(6) = 6$$

5. If A is a square matrix of order 3 such that $\det(A) = 3$ and $\det(\text{adj}(-4 \text{adj}(-3 \text{adj}(3 \text{adj}((2A)^{-1})))))) = 2^m 3^n$, then $m + 2n$ is equal to:

(1) 3

(2) 2

(3) 4

(4) 6

Ans. (3)

Sol.

$$|A| = 3$$

$$|\text{adj}(-4 \text{adj}(-3 \text{adj}(3 \text{adj}((2A)^T))))|$$

$$= |-4 \text{adj}(-3 \text{adj}(3 \text{adj}(2A)^T))|^2$$

$$= 4^6 |\text{adj}(-3 \text{adj}(3 \text{adj}(2A)^T))|^2$$

$$= 2^{12} \cdot 3^{12} \cdot |\text{adj}(2A)^T|^8$$

$$2^{12} \cdot 3^{12} \cdot 3^{24} |\text{adj}(2A)^T|^8$$

$$= 2^{12} \cdot 3^{36} \cdot |(2A)^{-1}|^{16}$$

$$= 2^{12} \cdot 3^{36} \cdot \frac{1}{|2A|^{16}}$$

$$= 2^{12} \cdot 3^{36} \cdot \frac{1}{2^{48} |\det A|^{16}}$$

$$= 2^{12} \cdot 3^{36} \cdot \frac{1}{2^{48} \cdot 3^{16}}$$

$$= \frac{3^{20}}{2^{36}} = 2^{-36} \cdot 3^{20}$$

6. Let $A = \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}$. If the sum of the diagonal elements of A^{13} is 3^n , then n is equal to _____.

Ans. (7)

Sol.

$$A = \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -3 \\ 3 & 0 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 3 & -3 \\ 3 & 0 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -6 \\ 6 & -3 \end{bmatrix}$$

$$A^4 = \begin{bmatrix} 3 & -6 \\ 6 & -3 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -9 \\ 9 & -9 \end{bmatrix}$$

$$A^5 = \begin{bmatrix} 0 & -9 \\ 9 & -9 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} -9 & -9 \\ 9 & -18 \end{bmatrix}$$

$$A^6 = \begin{bmatrix} -9 & -9 \\ 9 & -18 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} -27 & 0 \\ 0 & -27 \end{bmatrix}$$

$$A^7 = \begin{bmatrix} -27 & 0 & -27 \\ 0 & -27 & -27 \\ -54 & 27 & -27 \end{bmatrix} \begin{bmatrix} 3^6 \times 2 \\ 27^2 \\ -27^2 \end{bmatrix}$$

$$3^7 = 3^n \Rightarrow n = 7$$

7. Let $B = \begin{bmatrix} 1 & 3 \\ 1 & 5 \end{bmatrix}$ and A be a 2×2 matrix such that $AB^{-1} = A^{-1}$. If $BCB^{-1} = A$ and $C^4 + \alpha C^2 + \beta I = O$, then $2\beta - \alpha$ is equal to :

(1) 16

(2) 2

(3) 8

(4) 10

Ans. (4)

Sol. $BCB^{-1} = A$

$$\Rightarrow (BCB^{-1})(BCB^{-1}) = A.A$$

$$\Rightarrow BCICB^{-1} = A^2$$

$$\Rightarrow BC^2B^{-1} = A^2$$

$$\Rightarrow B^{-1}(BC^2B^{-1})B = B^{-1}(A.A)B$$

From equation (1)

$$C^2 = A^{-1}.A.B$$

$$C^2 = B$$

Also $AB^{-1} = A^{-1}$

$$\Rightarrow AB^{-1}.A = A^{-1}A = I$$

$$\Rightarrow A^{-1}(AB^{-1}A) = A^{-1}I$$

$$B^{-1}A = A^{-1}$$

Now characteristics equation of C^2 is

$$|C^2 - \lambda I| = 0$$

$$|B - \lambda I| = 0$$

$$\Rightarrow \begin{vmatrix} 1 - \lambda & 3 \\ 1 & 5 - \lambda \end{vmatrix} = 0$$

$$\Rightarrow (1 - \lambda)(5 - \lambda) - 3 = 0 \Rightarrow (\lambda^2 - 6\lambda + 5) - 3 = 0$$

$$\Rightarrow \lambda^2 - 6\lambda + 2 = 0$$

$$\Rightarrow \beta^2 - 6\beta + 2I = 0$$

$$\Rightarrow C^4 - 6C^2 + 2I = 0$$

$$\alpha = -6$$

$$\beta = 2$$

$$\therefore 2\beta - \alpha = 4 + 6 = 10$$

8. If $A = \begin{bmatrix} \sqrt{2} & 1 \\ -1 & \sqrt{2} \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$, $C = ABA^T$, $X = A^T C^2 A$. then, $\det X$ is equal to :

(1) 243

(2) 729

(3) 27

(4) 891

Ans. (2)

Sol.

$$A = \begin{bmatrix} \sqrt{2} & 1 \\ -1 & \sqrt{2} \end{bmatrix} \Rightarrow \det(A) = 3$$

$$B = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \Rightarrow \det(B) = 1$$

$$\text{Now } C = ABA^T \Rightarrow \det(C) = (\det(A))^2 \times \det(B)$$

$$|C| = 9$$

$$\text{Now } |X| = |A^T C^2 A|$$

$$= |A^T| \cdot |C^2| \cdot |A|$$

$$= |A|^2 |C|^2$$

$$= 9 \times 81$$

$$= 729$$

9. Consider the matrix $f(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$. Given below are two

statements :

Statement I: $f(-x)$ is the inverse of the matrix $f(x)$.

Statement II: $f(x) f(y) = f(x + y)$.

In the light of the above statements, choose the correct answer from the options given below

(1) Statement I is false but Statement II is true

(2) Both Statement I and Statement II are false

(3) Statement I is true but Statement II is false

(4) Both Statement I and Statement II are true

Ans. (4)

Sol.

$$f(-x) = \begin{bmatrix} \cos x & \sin x & 0 \\ -\sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$f(x) \cdot f(-x) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$

$$f(y) = \begin{bmatrix} \cos y & -\sin y & 0 \\ \sin y & \cos y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$f(x) \cdot f(y) = \begin{bmatrix} \cos(x+y) & -\sin(x+y) & 0 \\ \sin(x+y) & \cos(x+y) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

10. $A = \begin{bmatrix} 2 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$ Let, $B = [B_1, B_2, B_3]$, where B_1, B_2, B_3 are column matrices, and

$$AB_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad AB_2 = \begin{bmatrix} 3 \\ 0 \\ 0 \end{bmatrix}, \quad AB_3 = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$$

If $\alpha = |B|$ and β is the sum of all the diagonal elements of B , then $\alpha^3 + \beta^3$ is equal to _____.

Ans. (28)

Sol.

$$A = \begin{bmatrix} 2 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}, \quad B = [B_1, B_2, B_3]$$

$$B_1 = \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix}, \quad B_2 = \begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix}, \quad B_3 = \begin{bmatrix} x_3 \\ y_3 \\ z_3 \end{bmatrix}$$

$$AB_1 = \begin{bmatrix} 2 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$x_1 = 1, \quad y_1 = -1, \quad z_1 = -1$$

$$AB_2 = \begin{bmatrix} 2 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \\ 0 \end{bmatrix}$$

$$x_2 = 2, \quad y_2 = 1, \quad z_2 = -2$$

$$AB_3 = \begin{bmatrix} 2 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_3 \\ y_3 \\ z_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$$

$$x_3 = 2, \quad y_3 = 0, \quad z_3 = -1$$

$$B = \begin{bmatrix} 1 & 2 & 2 \\ -1 & 1 & 0 \\ -1 & -2 & -1 \end{bmatrix}$$

$$\alpha = |B| = 3, \quad \beta = 1$$

$$\alpha^3 + \beta^3 = 27 + 1 = 28$$

11. Let A be a 3×3 real matrix such that

$$A \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = 2 \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \quad A \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} = 4 \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}, \quad A \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} = 2 \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$$

$$(A - 3I) \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

(1) unique solution

(2) exactly two solutions

(3) no solution

(4) infinitely many solutions

Ans. (1)

Sol.-

$$A = \begin{bmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{bmatrix}$$

$$A \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 2 \end{bmatrix}$$

$$\text{Given } \begin{bmatrix} x_1 + z_1 \\ x_2 + z_2 \\ x_3 + z_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 2 \end{bmatrix}$$

$$\therefore x_1 + z_1 = 2 \dots (2)$$

$$x_2 + z_2 = 0 \dots (3)$$

$$x_3 + z_3 = 0 \dots (4)$$

$$A = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}, \quad \text{Given } \begin{bmatrix} -x_1 + z_1 \\ -x_2 + z_2 \\ -x_3 + z_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \\ 4 \end{bmatrix}$$

$$\Rightarrow -x_1 + z_1 = 4 \dots (5)$$

$$-x_2 + z_2 = 0 \dots (6)$$

$$-x_3 + z_3 = 4 \dots (7)$$

$$A = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \quad \text{Given } \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}$$

$$\therefore y_1 = 0, \quad y_2 = 2, \quad y_3 = 0$$

\therefore from (2), (3), (4), (5), (6), and (7):

$$x_1 = 3, \quad x_2 = 0, \quad x_3 = -1$$

$$y_1 = 0, \quad y_2 = 2, \quad y_3 = 0$$

$$z_1 = -1, \quad z_2 = 0, \quad z_3 = 3$$

$$\therefore A = \begin{bmatrix} 3 & 0 & -1 \\ 3 & 2 & 0 \\ -1 & 0 & 3 \end{bmatrix}$$

$$\therefore (A - 3I) \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & -1 \\ -1 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -z \\ -y \\ -x \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ -3 \end{bmatrix} \Rightarrow [z = -1], \quad [y = -2], \quad [x = -3]$$

$$2^{2024} = 9m + 4, \quad m \leftarrow \text{even}$$

$$2^{9m+4} = 16 \cdot (2^3)^{3m} \equiv 16 \pmod{9}$$

Previous Year Questions from Determinants

1. If the system of equations

$$x + (\sqrt{2} \sin \alpha)y + (\sqrt{2} \cos \alpha)z = 0$$

$$x + (\cos \alpha)y + (\sin \alpha)z = 0$$

$$x + (\sin \alpha)y - (\cos \alpha)z = 0$$

has a non-trivial solution, then $\alpha \in (0, \frac{\pi}{2})$ is equal to :

(1) $\frac{3\pi}{4}$

(2) $\frac{7\pi}{24}$

(3) $\frac{5\pi}{24}$

(4) $\frac{11\pi}{24}$

Ans. (3)

Sol.

$$\begin{vmatrix} 1 & \sqrt{2} \sin \alpha & \sqrt{2} \cos \alpha \\ 1 & \sin \alpha & -\cos \alpha \\ 1 & \cos \alpha & \sin \alpha \end{vmatrix} = 0$$

$$\Rightarrow 1 - \sqrt{2} \sin \alpha (\sin \alpha + \cos \alpha) + \sqrt{2} \cos \alpha (\cos \alpha - \sin \alpha) = 0$$

$$\Rightarrow 1 + \sqrt{2} \cos 2\alpha - \sqrt{2} \sin 2\alpha = 0$$

$$\cos 2\alpha - \sin 2\alpha = -\frac{1}{\sqrt{2}}$$

$$\cos \left(2\alpha + \frac{\pi}{4}\right) = -\frac{1}{2}$$

$$2\alpha + \frac{\pi}{4} = 2n\pi \pm \frac{2\pi}{3}$$

$$\alpha + \frac{\pi}{8} = n\pi \pm \frac{\pi}{3}$$

$$n = 0,$$

$$x = \frac{\pi}{3} - \frac{\pi}{8} = \frac{5\pi}{24}$$

2. If the system of equations

$$11x + y + \lambda z = -5$$

$$2x + 3y + 5z = 3$$

$$8x - 19y - 39z = \mu$$

has infinitely many solutions, then $\lambda^4 - \mu$ is equal to :

(1) 49

(2) 45

(3) 47

(4) 51

Ans. (3)

Sol.

$$11x + y + \lambda z = -5$$

$$2x + 3y + 5z = 3$$

$$8x - 19y - 39z = \mu$$

$$D = \begin{vmatrix} 11 & 1 & \lambda \\ 2 & 3 & 5 \\ 8 & -19 & -39 \end{vmatrix} = 0$$

$$\Rightarrow 11(-117 + 95) - 1(-78 - 40) + \lambda(-38 - 24)$$

$$\Rightarrow 11(-22) + 118 - \lambda(62) = 0$$

$$\Rightarrow -242 + 118 - 62\lambda = 0 \Rightarrow 62\lambda = 118 - 242$$

$$\lambda = \frac{-124}{62} = -2$$

$$D_1 = \begin{vmatrix} -5 & 1 & -2 \\ 3 & 3 & 5 \\ \mu & -19 & -39 \end{vmatrix} = 0$$

$$\Rightarrow -5(-117 + 95) - 1(-117 - 5\mu) - 2(-57 - 3\mu) = 0$$

$$\Rightarrow 5(-22) + 117 + 5\mu + 114 + 6\mu = 0$$

$$\Rightarrow 11\mu = -110 - 231 = -341 \Rightarrow \mu = -31$$

$$\lambda^4 - \mu = (-2)^4 - (-31) = 16 + 31 = 47$$

3. For $\alpha, \beta \in \mathbb{R}$ and a natural number n , let $A_n = \begin{bmatrix} r & 1 & \frac{n^2 + \alpha}{2} \\ 2r & 2 & n^2 - \beta \\ 3r - 2 & 3 & \frac{n(3n - 1)}{2} \end{bmatrix}$. Then

$2A_{10} - A_8$ is

(1) $4\alpha + 2\beta$

(2) $2\alpha + 4\beta$

(3) $2n$

(4) 0

Ans. (1)

Sol.

$$A_r = \begin{vmatrix} r & 1 & \frac{n^2}{2} + \alpha \\ 2r & 2 & n^2 - \beta \\ 3r - 2 & 3 & \frac{n(3n - 1)}{2} \end{vmatrix}$$

$$2A_{10} - A_8 = \begin{vmatrix} 20 & 1 & \frac{n^2}{2} + \alpha \\ 40 & 2 & n^2 - \beta \\ 56 & 3 & \frac{n(3n - 1)}{2} \end{vmatrix} - \begin{vmatrix} 8 & 1 & \frac{n^2}{2} + \alpha \\ 16 & 2 & n^2 - \beta \\ 22 & 3 & \frac{n(3n - 1)}{2} \end{vmatrix}$$

$$\Rightarrow \begin{vmatrix} 12 & 1 & \frac{n^2}{2} + \alpha \\ 24 & 2 & n^2 - \beta \\ 34 & 3 & \frac{n(3n - 1)}{2} \end{vmatrix}$$

$$\Rightarrow \begin{vmatrix} 0 & 1 & \frac{n^2}{2} + \alpha \\ 0 & 2 & n^2 - \beta \\ -2 & 3 & \frac{n(3n - 1)}{2} \end{vmatrix}$$

$$\Rightarrow -2 \left((n^2 - \beta) - (n^2 + 2\alpha) \right)$$

$$\Rightarrow -2(-\beta - 2\alpha) \Rightarrow 4\alpha + 2\beta$$

4. Let $\alpha\beta\gamma = 45$; $\alpha, \beta, \gamma \in \mathbb{R}$. If $x(\alpha, 1, 2) + y(1, \beta, 2) + z(2, 3, \gamma) = (0, 0, 0)$ for some $x, y, z \in \mathbb{R}$, $xyz \neq 0$, then $6\alpha + 4\beta + \gamma$ is equal to _____

Ans. (55)

Sol.

$$\alpha\beta\gamma = 45, \quad \alpha, \beta, \gamma \in \mathbb{R}$$

$$x(\alpha, 1, 2) + y(1, \beta, 2) + z(2, 3, \gamma) = (0, 0, 0)$$

$$x, y, z \in \mathbb{R}, \quad xyz \neq 0$$

$$\Rightarrow \alpha x + y + 2z = 0$$

$$x + \beta y + 3z = 0$$

$$2x + 2y + \gamma z = 0$$

$$xyz \neq 0 \Rightarrow \text{non-trivial}$$

$$\begin{vmatrix} \alpha & 1 & 2 \\ 1 & \beta & 3 \\ 2 & 2 & \gamma \end{vmatrix} = 0$$

$$\Rightarrow \alpha(\beta\gamma - 6) - 1(\gamma - 6) + 2(2 - 2\beta) = 0$$

$$\Rightarrow \alpha\beta\gamma - 6\alpha - \gamma + 6 + 4 - 4\beta = 0$$

$$\Rightarrow 6\alpha + 4\beta + \gamma = 55$$

5. Let the system of equations $x + 2y + 3z = 5$, $2x + 3y + z = 9$, $4x + 3y + \lambda z = \mu$ have infinite number of solutions. Then $\lambda + 2\mu$ is equal to :

(1) 28

(2) 17

(3) 22

(4) 15

Ans. (2)

Sol.

$$x + 2y + 3z = 5$$

$$2x + 3y + z = 9$$

$$4x + 3y + \lambda z = \mu$$

For infinite solutions, the following must hold: $\Delta = \Delta_1 = \Delta_2 = \Delta_3 = 0$

$$\Delta = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 4 & 3 & \lambda \end{vmatrix} = 0 \Rightarrow \lambda = -13$$

$$\Delta_1 = \begin{vmatrix} 5 & 2 & 3 \\ 9 & 3 & 1 \\ \mu & 3 & -13 \end{vmatrix} = 0 \Rightarrow \mu = 15$$

$$\Delta_2 = \begin{vmatrix} 1 & 5 & 3 \\ 2 & 9 & 1 \\ 4 & 15 & -13 \end{vmatrix} = 0$$

$$\Delta_3 = \begin{vmatrix} 1 & 2 & 5 \\ 2 & 3 & 9 \\ 4 & 3 & 15 \end{vmatrix} = 0$$

for $\lambda = -13$, $\mu = 15$, the system of equations has infinite solutions.

$$\Rightarrow \lambda + 2\mu = -13 + 30 = 17$$

6. The values of α , for which
$$\begin{vmatrix} 1 & \frac{3}{2} & \alpha + \frac{3}{2} \\ 1 & \frac{1}{3} & \alpha + \frac{1}{3} \\ 2\alpha + 3 & 3\alpha + 1 & 0 \end{vmatrix} = 0$$
, lie in the interval

(1) $(-2, 1)$

(2) $(-3, 0)$

(3) $\left(-\frac{3}{2}, \frac{3}{2}\right)$

(4) $(0, 3)$

Ans. (2)

Sol.

$$\begin{vmatrix} 1 & \frac{3}{2} & \alpha + \frac{3}{2} \\ 1 & \frac{1}{3} & \alpha + \frac{1}{3} \\ 2\alpha + 3 & 3\alpha + 1 & 0 \end{vmatrix} = 0$$

$$\Rightarrow (2\alpha + 3) \begin{vmatrix} \frac{3}{2} & \alpha + \frac{3}{2} \\ 1 & \alpha + \frac{1}{3} \end{vmatrix}$$

$$- (3\alpha + 1) \begin{vmatrix} 1 & \alpha + \frac{3}{2} \\ 1 & \alpha + \frac{1}{3} \end{vmatrix} = 0$$

$$\Rightarrow (2\alpha + 3) \cdot \frac{7\alpha}{6} + (3\alpha + 1) \cdot \frac{-7}{6} = 0$$

$$\Rightarrow 2\alpha^2 + 3\alpha + 3\alpha + 1 = 0$$

$$\Rightarrow 2\alpha^2 + 6\alpha + 1 = 0$$

$$\Rightarrow \alpha = \frac{-3 + \sqrt{7}}{2}, \frac{-3 - \sqrt{7}}{2}$$

7. Consider the system of linear equations $x + y + z = 5$, $x + 2y + \lambda^2 z = 9$, $x + 3y + \lambda z = \mu$, where $\lambda, \mu \in \mathbb{R}$. Then, which of the following statement is NOT correct?

(1) System has infinite number of solution if $\lambda = 1$ and $\mu = 13$

(2) System is inconsistent if $\lambda = 1$ and $\mu \neq 13$

(3) System is consistent if $\lambda \neq 1$ and $\mu = 13$

(4) System has unique solution if $\lambda \neq 1$ and $\mu \neq 13$

Ans. (4)

Sol.

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & \lambda^2 \\ 1 & 3 & \lambda \end{vmatrix} = 0$$

$$\Rightarrow 2\lambda^2 - \lambda - 1 = 0$$

$$\lambda = 1, \quad -\frac{1}{2}$$

$$\begin{vmatrix} 1 & 1 & 5 \\ 2 & \lambda^2 & 9 \\ 3 & \lambda & \mu \end{vmatrix} = 0 \Rightarrow \mu = 13$$

Infinite solution $\lambda = 1$ $\mu = 13$

For unique soln $\lambda \neq 1$

For no soln $\lambda = 1$ $\mu \neq 13$

If $\lambda \neq 1$ and $\mu \neq 13$

Considering the case when $\lambda = -\frac{1}{2}$ and $\mu \neq 13$ this will generate no solution ca

8. If then $f(x) = \begin{vmatrix} 2 \cos^4 x & 2 \sin^4 x & 3 + \sin^2 2x \\ 3 + 2 \cos^4 x & 2 \sin^4 x & \sin^2 2x \\ 2 \cos^4 x & 3 + 2 \sin^4 x & \sin^2 2x \end{vmatrix} \Rightarrow \frac{1}{5} f'(0)$ is equal to

(1) 0

(2) 1

(3) 2

(4) 6

Ans. (1)

Sol.

$$\begin{vmatrix} 2 \cos^4 x & 2 \sin^4 x & 3 + \sin^2 2x \\ 3 + 2 \cos^4 x & 2 \sin^4 x & \sin^2 2x \\ 2 \cos^4 x & 3 + 2 \sin^4 x & \sin^2 2x \end{vmatrix}$$

Row operations : $R_2 \rightarrow R_2 - R_1, R_3 \rightarrow R_3 - R_1$

$$\Rightarrow \begin{vmatrix} 2 \cos^4 x & 2 \sin^4 x & 3 + \sin^2 2x \\ 3 & 0 & -3 \\ 0 & 3 & -3 \end{vmatrix}$$

$$f(x) = 45$$

$$f'(x) = 0$$

9. If the system of linear equations

$$x - 2y + z = -4$$

$$2x + \alpha y + 3z = 5$$

$$3x - y + \beta z = 3$$

has infinitely many solutions, then $12\alpha + 13\beta$ is equal to

(1) 60

(2) 64

(3) 54

(4) 58

Ans. (4)

Sol.

$$D = \begin{vmatrix} 1 & -2 & 1 \\ 2 & \alpha & 3 \\ 3 & -1 & \beta \end{vmatrix}$$

$$= 1(\alpha\beta + 3) + 2(2\beta - 9) + 1(-2 - 3\alpha)$$

$$= \alpha\beta + 3 + 4\beta - 18 - 2 - 3\alpha$$

$$= \alpha\beta - 3\alpha + 4\beta - 17 \dots (1)$$

$$D_1 = \begin{vmatrix} -4 & -2 & 1 \\ 5 & \alpha & 3 \\ 3 & -1 & \beta \end{vmatrix} = 0$$

$$D_2 = \begin{vmatrix} 1 & -4 & 1 \\ 2 & 5 & 3 \\ 3 & 3 & \beta \end{vmatrix} = 0$$

$$\Rightarrow 1(5\beta - 9) + 4(2\beta - 9) + 1(6 - 15) = 0$$

$$5\beta - 9 + 8\beta - 36 - 9 = 0$$

$$13\beta = 54 \Rightarrow \beta = \frac{54}{13} \quad \text{put in (1)}$$

$$\frac{54}{13}\alpha - 3\alpha + 4\left(\frac{54}{13}\right) = 17$$

$$\frac{54\alpha}{13} - \frac{39\alpha}{13} + \frac{216}{13} = 17$$

$$\frac{15\alpha + 216}{13} = 17 \Rightarrow 15\alpha = 5 \Rightarrow \alpha = \frac{1}{3}$$

$$\text{Now, } 12\alpha + 13\beta = 12 \cdot \frac{1}{3} + 13 \cdot \frac{54}{13}$$

$$= 4 + 54 = 58$$