PERMUTATION & COMBINATION

1. Let $\alpha = \frac{(4!)!}{(4!)^{3!}}$ and $\beta = \frac{(5!)!}{(5!)^{4!}}$. Then :

(1) $\alpha \in N$ and $\beta \notin N$ (2) $\alpha \notin N$ and $\beta \in N$

(3) $\alpha \in N$ and $\beta \in N$ (4) $\alpha \notin N$ and $\beta \notin N$

2. All the letters of the word "GTWENTY" are written in all possible ways with or without meaning and these words are written as in a dictionary. The serial number of the word "GTWENTY" IS

3. Number of ways of arranging 8 identical books into 4 identical shelves where any number of shelves may remain empty is equal to

(1) 18 (2) 16

(3) 12 (4) 15

4. In an examination of Mathematics paper, there are 20 questions of equal marks and the question paper is divided into three sections : A, B and C . A student is required to attempt total 15 questions taking at least 4 questions from each section. If section A has 8 questions, section B has 6 questions and section C has 6 questions, then the total number of ways a student can select 15 questions is _______.

5. The total number of words (with or without meaning) that can be formed out of the letters of the word 'DISTRIBUTION' taken four at a time, is equal to _____

6. The number of ways in which 21 identical apples can be distributed among three children such that each child gets at least 2 apples, is

(1) 406

(2) 130

(3) 142

(4) 136

7. If for some m, n; ${}^6C_m + 2\left({}^6C_{m+1}\right) + {}^6C_{m+2} > {}^8C_3$ and ${}^{n-1}P_3 : {}^nP_4 = 1 : 8$, then ${}^nP_{m+1} + {}^{n+1}C_m$ is equal to

(1) 380 (2) 376

(3) 384 (4) 372

8. If n is the number of ways five different employees can sit into four indistinguishable offices where any office may have any number of persons including zero, then n is equal to:

(1) 47 (2) 53

(3) 51 (4) 43

9. The number of elements in the set

$S = \{(x, y, z) : x, y, z \in \mathbf{Z}, x + 2y + 3z = 42, x, y, z \ge 0\}$

equals __________.

10. The lines $L_1, L_2, \ldots, L_{20}$ are distinct. For n = 1, 2, 3, …, 10 all the lines L_{2n-1} are parallel to each other and all the lines L_{2n} pass through a given point P. The maximum number of points of intersection of pairs of lines from the set $\{L_1, L_2, \ldots, L_{20}\}$ is equal to :

11. The number of ways five alphabets can be chosen from the alphabets of the word MATHEMATICS, where the chosen alphabets are not necessarily distinct, is equal to :

(1) 175 (2) 181

(3) 177 (4) 179

12. There are 5 points P_1, P_2, P_3, P_4, P_5 on the side AB, excluding A and B, of a triangle ABC. Similarly there are 6 points $P_6, P_7, \ldots, P_{11}$ on the side BC and 7 points $P_{12}, P_{13}, \ldots, P_{18}$ on the side CA of the triangle. The number of triangles, that can be formed using the points $P_1, P_2, \ldots, P_{18}$ as vertices, is :

(1) 776 (2) 751

(3) 796 (4) 771

13. 60 words can be made using all the letters of the word BHBJO, with or without meaning. If these words are written as in a dictionary, then the 50^{th} word is :

(1) OBBHJ (2) HBBJO
(3) OBBJH (4) JBBOH

14. Let the set S = {2, 4, 8, 16,....., 512} be partitioned into 3 sets A, B, C with equal number of elements such that $A \cup B \cup C = S$ and $A \cap B = B \cap C = A \cap C = \varphi$. The maximum number of such possible partitions of S is equal to :

(1) 1680 (2) 1520 (3) 1710 (4) 1640

15. There are 4 men and 5 women in Group A, and 5 men and 4 women in Group B. If 4 persons are selected from each group, then the number of ways of selecting 4 men and 4 women is …..

16. If all the words with or without meaning made using all the letters of the word "NAGPUR" are arranged as in a dictionary, then the word at 315^{th} position in this arrangement is :

(1) NRAGUP (2) NRAGPU
(3) NRAPGU (4) NRAPUG

17. The number of ways of getting a sum 16 on throwing a dice four times is __________.

18. The number of triangles whose vertices are at the vertices of a regular octagon but none of whose sides is a side of the octagon is

(1) 24 (2) 56 (3) 16 (4) 48

19. The number of 3-digit numbers, formed using the digits 2, 3, 4, 5 and 7, when the repetition of digits is not allowed, and which are not divisible by 3, is equal to ______.

SOLUTIONS

1. Ans. (3)

Sol. $\alpha = \dfrac{(4!)!}{(4!)^{3!}}, \beta = \dfrac{(5!)!}{(5!)^{4!}}$

$\alpha = \dfrac{(24)!}{(4!)^{6}}, \beta = \dfrac{(120)!}{(5!)^{24}}$

Let 24 distinct objects are divided into 6 groups of 4 objects in each group.

No. of ways of formation of group $= \dfrac{24!}{(4!)^6 . 6!} \in N$

Similarly,

Let 120 distinct objects are divided into 24 groups of 5 objects in each group.

No. of ways of formation of groups

$= \dfrac{(120)!}{(5!)^{24} . 24!} \in N$

2. Ans. (553)

Sol. Words starting with E = 360

Words starting with GE = 60

Words starting with GN = 60

Words starting with GTE = 24

Words starting with GTN = 24

Words starting with GTT = 24

GTWENTY = 1

Total = 553

3. Ans. (4)

Sol. 3 Shelf empty : (8, 0, 0, 0) → 1way

2 shelf empty : (7,1, 0, 0), (6, 2, 0, 0), (5, 3, 0, 0), (4, 4, 0, 0) → 4ways

1 shelf empty : (6,1,1, 0), (3, 3, 2, 0), (5, 2,1, 0), (4, 2, 2, 0), (4, 3,1, 0) → 5ways

0 Shelf empty : (1, 2, 3, 2), (5,1,1,1), (2, 2, 2, 2), (3, 3,1,1), (4, 2,1,1) → 5ways

Total = 15 ways

4. Ans. (11376)

Sol. If 4 questions from each section are selected

Remaining 3 questions can be selected either in (1, 1, 1) or (3, 0, 0) or (2, 1, 0)

$\therefore$ Total ways $= {}^8c_5 . {}^6c_5 . {}^6c_5 + {}^8c_6 \, {}^6c_5 . {}^6c_4 . 2$

$+ {}^8c_5 . {}^6c_6 . {}^6c_4 . 2 + {}^8c_4 . {}^6c_6 . {}^6c_5 . 2 + {}^8c_7 . {}^6c_4 . {}^6c_4$

= 56 . 6 . 6 + 28 . 6 . 15 . 2 + 56 . 15 . 2 + 70 . 6 . 2 + 8 .

= 2016 + 5040 + 1680 + 840 + 1800 = 11376

5. Ans. (3734)

Sol. We have III, TT, D, S, R, B, U, O, N

Number of words with selection (a, a, a, b)

$= {}^8C_1 . \dfrac{4!}{3!} = 32$

Number of words with selection (a, a, b, b)

$= \dfrac{4!}{2!2!} = 6$

Number of words with selection (a, a, b, c)

$= {}^2C_1 . {}^8C_2 . \dfrac{4!}{2!} = 672$

Number of words with selection (a, b, c, d)

$= {}^9C_4 . 4! = 3024$

$\therefore$ total = 3024 + 672 + 6 + 32

= 3734

6. Ans. (4)

Sol. After giving 2 apples to each child 15 apples left now 15 apples can be distributed in

${}^{15+3-1}C_2 = {}^{17}C_2$ ways

$= \dfrac{17 . 16}{2} = 136$

7. Ans. (4)

Sol. $^6C_m + 2\left(^6C_{m+1}\right) + ^6C_{m+2} > ^8C_3$

$^7C_{m+1} + ^7C_{m+2} > ^8C_3$

$^8C_{m+2} > ^8C_3$

$\therefore m = 2$

And $^{n-1}P_3 : {}^nP_4 = 1 : 8$

$$\frac{(n-1)(n-2)(n-3)}{n(n-1)(n-2)(n-3)} = \frac{1}{8}$$

$\therefore n = 8$

$\therefore {}^nP_{m+1} + {}^{n+1}C_m = {}^8P_3 + {}^9C_2$

$= 8 \cdot 7 \cdot 6 + \frac{9 \cdot 8}{2}$

= 372

8. Ans. (3)

Sol. Total ways to partition 5 into 4 parts are :

5, 0, 0, 0 $\Rightarrow$ 1 way

4, 1, 0, 0 $\Rightarrow \frac{5!}{4!} = 5$ ways

3, 2, 0, 0, $\Rightarrow \frac{5!}{3!2!} = 10$ ways

2, 2, 0,1 $\Rightarrow \frac{5!}{2!2!2!} = 15$ ways

2,1,1,1 $\Rightarrow \frac{5!}{2!(1!)^3 3!} = 10$ ways

3,1,1, 0 $\Rightarrow \frac{5!}{3!2!} = 10$ ways

Total $\Rightarrow$ 1+5+10+15+10+10 = 51 ways

9. Ans. (169)

Sol. $x + 2y + 3z = 42$, $x, y, z \geq 0$

$z = 0$	$x + 2y = 42 \Rightarrow 22$
$z = 1$	$x + 2y = 39 \Rightarrow 20$
$z = 2$	$x + 2y = 36 \Rightarrow 19$
$z = 3$	$x + 2y = 33 \Rightarrow 17$
$z = 4$	$x + 2y = 30 \Rightarrow 16$
$z = 5$	$x + 2y = 27 \Rightarrow 14$
$z = 6$	$x + 2y = 24 \Rightarrow 13$
$z = 7$	$x + 2y = 21 \Rightarrow 11$
$z = 8$	$x + 2y = 18 \Rightarrow 10$
$z = 9$	$x + 2y = 15 \Rightarrow 8$
$z = 10$	$x + 2y = 12 \Rightarrow 7$
$z = 11$	$x + 2y = 9 \Rightarrow 5$
$z = 12$	$x + 2y = 6 \Rightarrow 4$
$z = 13$	$x + 2y = 3 \Rightarrow 2$
$z = 14$	$x + 2y = 0 \Rightarrow 1$

Total : 169

10. Ans. (101)

Sol. $L_1, L_3, L_5, \ldots L_{19}$ are Parallel

$L_2, L_4, L_6, \ldots L_{20}$ are Concurrent

Total points of intersection $= {}^{20}C_2 - {}^{10}C_2 - {}^{10}C_2 + 1$

= 101

11. Ans. (4)

Sol. AA, MM, TT, H, I, C, S, E

(1) All distinct

$^8C_5 \rightarrow 56$

(2) 2 same, 3 different

$^3C_1 \times {}^7C_3 \rightarrow 105$

(3) 2 same 1st kind, 2 same 2nd kind, 1 different

$^3C_2 \times {}^6C_1 \rightarrow 18$

Total $\rightarrow$ 179

12. Ans. (2)

Sol. $^{18}C_3 - {}^5C_3 - {}^6C_3 - {}^7C_3$

= 751

13. Ans. (3)

Sol. B B H J O

[B] ______ $4! = 24$

[H] ______ $\frac{4!}{2!} = 12$

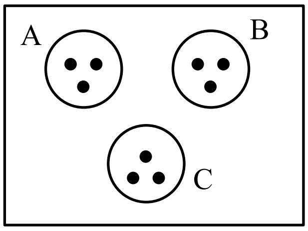
[J] ______ $\frac{4!}{2!} = 12$

O B B H J

O B B J H → 50^{th} rank

14. Ans. (1)

Sol.



$$\frac{9!}{(3!3!3!)3!} \cdot 3!$$

15. Ans. (5626)

Sol.

From Group A	From Group B	Ways of selection
4M	4W	${}^4C_4 \times {}^4C_4 = 1$
3M 1W	1M 3W	${}^4C_3 \times {}^5C_1 \times {}^5C_1 \times {}^4C_3 = 400$
2M 2W	2M 2W	${}^4C_2 \times {}^5C_2 \times {}^5C_2 \times {}^4C_2 = 3600$
1M 3W	3M 1W	${}^4C_1 \times {}^5C_3 \times {}^5C_3 \times {}^4C_1 = 1600$
4W	4M	${}^5C_4 \times {}^5C_4 = 25$
Total		5626

16. Ans. (3)

Sol. NAGPUR

A → 5! = 120

G → 5! = 120   240

NA → 4! = 24   264

NG → 4! = 24   288

NP → 4! = 24   312

NRAGPU = 1   313

NRAGUP   314

NRAPGU   315

17. Ans. (125)

Sol. $(x^1 + x^2 \ldots + x^6)^4$

$$x^4 \cdot \left(\frac{1-x^6}{1-x}\right)^4$$

$x^4 \cdot (1-x^6)^4 \cdot (1-x)^{-4}$

$x^4[1-4x^6 + 6x^{12} \ldots] [(1-x)^{-4}]$

$(x^4 - 4x^{10} + 6x^{16} \ldots) (1-x)^{-4}$

$(x^4 - 4x^{10} + 6x^{16}) (1 + {}^{15}C_{12} x^{12} + {}^9C_6 x^6 \ldots)$

$({}^{15}C_{12} - 4 \cdot {}^9C_6 + 6)x^{16}$

$({}^{15}C_3 - 4 \cdot {}^9C_6 + 6)$

$= 35 \times 13 - 6 \times 8 \times 7 + 6$

$= 455 - 336 + 6$

$= 125$

18. Ans. (3)

Sol. ∵ no. of triangles having no side common with a n sided polygon $= \frac{{}^nC_1 \cdot {}^{n-4}C_2}{3}$

$$= \frac{{}^8C_1 \cdot {}^4C_2}{3} = 16$$

19. Ans. (36)

Sol. **2, 3, 4, 5, 7**

total number of three digit numbers not divisible by 3 will be formed by using the digits

(4, 5, 7)

(3, 4, 7)

(2, 5, 7)

(2, 4, 7)

(2, 4, 5)

(2, 3, 5)

number of ways = 6 × 3! = 36