PROBABILITY

1. A fair die is tossed repeatedly until a six is obtained. Let X denote the number of tosses required and let a = P(X = 3), b = P(X≥ 3) and c = P(X ≥ 6 |X > 3). Then $\frac{b+c}{a}$ is equal to ______.

2. An urn contains 6 white and 9 black balls. Two successive draws of 4 balls are made without replacement. The probability, that the first draw gives all white balls and the second draw gives all black balls, is :

(1) $\frac{5}{256}$ (2) $\frac{5}{715}$

(3) $\frac{3}{715}$ (4) $\frac{3}{256}$

3. A fair die is thrown until 2 appears. Then the probability, that 2 appears in even number of throws, is

(1) $\frac{5}{6}$ (2) $\frac{1}{6}$ (3) $\frac{5}{11}$ (4) $\frac{6}{11}$

4. An integer is chosen at random from the integers 1, 2, 3, …, 50. The probability that the chosen integer is a multiple of atleast one of 4, 6 and 7 is

(1) $\frac{8}{25}$ (2) $\frac{21}{50}$ (3) $\frac{9}{50}$ (4) $\frac{14}{25}$

5. Two integers x and y are chosen with replacement from the set {0, 1, 2, 3, ….., 10}. Then the probability that $|x-y|>5$ is :

(1) $\frac{30}{121}$ (2) $\frac{62}{121}$

(3) $\frac{60}{121}$ (4) $\frac{31}{121}$

6. Bag A contains 3 white, 7 red balls and bag B contains 3 white, 2 red balls. One bag is selected at random and a ball is drawn from it. The probability of drawing the ball from the bag A, if the ball drawn in white, is :

(1) $\frac{1}{4}$ (2) $\frac{1}{9}$ (3) $\frac{1}{3}$ (4) $\frac{3}{10}$

7. Two marbles are drawn in succession from a box containing 10 red, 30 white, 20 blue and 15 orange marbles, with replacement being made after each drawing. Then the probability, that first drawn marble is red and second drawn marble is white, is

(1) $\frac{2}{25}$ (2) $\frac{4}{25}$ (3) $\frac{2}{3}$ (4) $\frac{4}{75}$

8. A coin is based so that a head is twice as likely to occur as a tail. If the coin is tossed 3 times, then the probability of getting two tails and one head is -

(1) $\frac{2}{9}$ (2) $\frac{1}{9}$ (3) $\frac{2}{27}$ (4) $\frac{1}{27}$

9. A bag contains 8 balls, whose colours are either white or black. 4 balls are drawn at random without replacement and it was found that 2 balls are white and other 2 balls are black. The probability that the bag contains equal number of white and black balls is:

(1) $\frac{2}{5}$ (2) $\frac{2}{7}$ (3) $\frac{1}{7}$ (4) $\frac{1}{5}$

10. Let Ajay will not appear in JEE exam with probability $p=\frac{2}{7}$, while both Ajay and Vijay will appear in the exam with probability $q=\frac{1}{5}$. Then the probability, that Ajay will appear in the exam and Vijay will not appear is :

(1) $\frac{9}{35}$ (2) $\frac{18}{35}$ (3) $\frac{24}{35}$ (4) $\frac{3}{35}$

11. There are three bags X, Y and Z. Bag X contains 5 one-rupee coins and 4 five-rupee coins; Bag Y contains 4 one-rupee coins and 5 five-rupee coins and Bag Z contains 3 one-rupee coins and 6 five-rupee coins. A bag is selected at random and a coin drawn from it at random is found to be a one-rupee coin. Then the probability, that it came from bag Y, is :

(1) $\frac{1}{3}$ (2) $\frac{1}{2}$ (3) $\frac{1}{4}$ (4) $\frac{5}{12}$

12. If an unbiased dice is rolled thrice, then the probability of getting a greater number in the i^{th} roll than the number obtained in the $(i-1)^{th}$ roll, $i = 2, 3$, is equal to :

(1) 3/54 (2) 2/54
(3) 5/54 (4) 1/54

13. Let a, b and c denote the outcome of three independent rolls of a fair tetrahedral die, whose four faces are marked 1, 2, 3, 4. If the probability that $ax^2 + bx + c = 0$ has all real roots is $\frac{m}{n}$, gcd(m, n) = 1, then m + n is equal to ________.

14. Three urns A, B and C contain 7 red, 5 black; 5 red, 7 black and 6 red, 6 black balls, respectively. One of the urn is selected at random and a ball is drawn from it. If the ball drawn is black, then the probability that it is drawn from urn A is :

(1) $\frac{4}{17}$ (2) $\frac{5}{18}$
(3) $\frac{7}{18}$ (4) $\frac{5}{16}$

15. In a tournament, a team plays 10 matches with probabilities of winning and losing each match as $\frac{1}{3}$ and $\frac{2}{3}$ respectively. Let x be the number of matches that the team wins, and y be the number of matches that team loses. If the probability $P(|x-y| \le 2)$ is p, then 3^9p equals……

16. The coefficients a, b, c in the quadratic equation $ax^2 + bx + c = 0$ are from the set {1, 2, 3, 4, 5, 6}. If the probability of this equation having one real root bigger than the other is p, then 216p equals :

(1) 57 (2) 38
(3) 19 (4) 76

17. If three letters can be posted to any one of the 5 different addresses, then the probability that the three letters are posted to exactly two addresses is:

(1) $\frac{12}{25}$ (2) $\frac{18}{25}$ (3) $\frac{4}{25}$ (4) $\frac{6}{25}$

18. The coefficients a, b, c in the quadratic equation $ax^2 + bx + c = 0$ are chosen from the set {1, 2, 3, 4, 5, 6, 7, 8}. The probability of this equation having repeated roots is :

(1) $\frac{3}{256}$ (2) $\frac{1}{128}$ (3) $\frac{1}{64}$ (4) $\frac{3}{128}$

19. Let the sum of two positive integers be 24. If the probability, that their product is not less than $\frac{3}{4}$ times their greatest positive product, is $\frac{m}{n}$, where gcd(m, n) = 1, then n – m equals :

(1) 9 (2) 11
(3) 8 (4) 10

20. A company has two plants A and B to manufacture motorcycles. 60% motorcycles are manufactured at plant A and the remaining are manufactured at plant B. 80% of the motorcycles manufactured at plant A are rated of the standard quality, while 90% of the motorcycles manufactured at plant B are rated of the standard quality. A motorcycle picked up randomly from the total production is found to be of the standard quality. If p is the probability that it was manufactured at plant B, then 126p is

(1) 54 (2) 64
(3) 66 (4) 56

21. Three balls are drawn at random from a bag containing 5 blue and 4 yellow balls. Let the random variables X and Y respectively denote the number of blue and Yellow balls. If $\bar{X}$ and $\bar{Y}$ are the means of X and Y respectively, then $7\bar{X} + 4\bar{Y}$ is equal to ______.

SOLUTIONS

1. Ans. (12)

Sol. $a = P(X = 3) = \frac{5}{6}\cdot\frac{5}{6}\cdot\frac{1}{6} = \frac{25}{216}$

$b = P(X \ge 3) = \frac{5}{6}\cdot\frac{5}{6}\cdot\frac{1}{6} + \left(\frac{5}{6}\right)^3\cdot\frac{1}{6}$

$+\left(\frac{5}{6}\right)^4\cdot\frac{1}{6} + \ldots\ldots$

$= \frac{\frac{25}{216}}{1-\frac{5}{6}} = \frac{25}{216}\cdot\frac{6}{1} = \frac{25}{36}$

$P(X \ge 6) = \left(\frac{5}{6}\right)^5\cdot\frac{1}{6} + \left(\frac{5}{6}\right)^6\cdot\frac{1}{6} + \ldots\ldots.$

$= \frac{\left(\frac{5}{6}\right)^5\cdot\frac{1}{6}}{1-\frac{5}{6}} = \left(\frac{5}{6}\right)^5$

$c = \frac{\left(\frac{5}{6}\right)^5}{\left(\frac{5}{6}\right)^3} = \frac{25}{36}$

$\frac{b+c}{a} = \frac{\left(\frac{5}{6}\right)^2 + \left(\frac{5}{6}\right)^2}{\left(\frac{5}{6}\right)^2\cdot\frac{1}{6}} = 12$

2. Ans. (3)

Sol. $\frac{{}^6C_4}{{}^{15}C_4}\cdot\frac{{}^9C_4}{{}^{11}C_4} = \frac{3}{715}$

Hence option (3) is correct.

3. Ans. (3)

Sol. Required probability =

$\frac{5}{6}\cdot\frac{1}{6} + \left(\frac{5}{6}\right)^3\cdot\frac{1}{6} + \left(\frac{5}{6}\right)^5\cdot\frac{1}{6} + \ldots..$

$= \frac{1}{6}\cdot\frac{\frac{5}{6}}{1-\frac{25}{36}} = \frac{5}{11}$

4. Ans. (2)

Sol. Given set = {1, 2, 3, …….. 50}

P(A) = Probability that number is multiple of 4

P(B) = Probability that number is multiple of 6

P(C) = Probability that number is multiple of 7

Now,

$P(A) = \frac{12}{50},\ P(B) = \frac{8}{50},\ P(C) = \frac{7}{50}$

again

$P(A\cap B) = \frac{4}{50},\ P(B\cap C) = \frac{1}{50},\ P(A\cap C) = \frac{1}{50}$

$P(A\cap B\cap C) = 0$

Thus

$P(A\cup B\cup C) = \frac{12}{50}+\frac{8}{50}+\frac{7}{50}-\frac{4}{50}-\frac{1}{50}-\frac{1}{50}+0$

$= \frac{21}{50}$

5. Ans. (1)

Sol. If $x = 0$, $y = 6, 7, 8, 9, 10$

If $x = 1$, $y = 7, 8, 9, 10$

If $x = 2$, $y = 8, 9, 10$

If $x = 3$, $y = 9, 10$

If $x = 4$, $y = 10$

If $x = 5$, y = no possible value

Total possible ways = (5 + 4 + 3 + 2 + 1) × 2 = 30

Required probability = $\frac{30}{11\cdot 11} = \frac{30}{121}$

6. Ans. (3)

Sol. E_1 : A is selected

A	B
3W 7R	3W 2 R

E_2 : B is selected

E : white ball is drawn

$P(E_1/E) = \frac{P(E).P(E/E_1)}{P(E_1).P(E/E_1) + P(E_2).P(E/E_2)}$

$= \frac{\frac{1}{2}\cdot\frac{3}{10}}{\frac{1}{2}\cdot\frac{3}{10}+\frac{1}{2}\cdot\frac{3}{5}}$

$= \frac{3}{3+6} = \frac{1}{3}$

7. Ans. (4)

Sol. Probability of drawing first red and then white

$= \frac{10}{75}\cdot\frac{30}{75} = \frac{4}{75}$

8. Ans. (1)

Sol. Let probability of tail is $\frac{1}{3}$

$\Rightarrow$ Probability of getting head = $\frac{2}{3}$

$\therefore$ Probability of getting 2 tails and 1 head

$$=\left(\frac{1}{3}\cdot\frac{2}{3}\cdot\frac{1}{3}\right)\cdot 3$$

$$=\frac{2}{27}\cdot 3 = \frac{2}{9}$$

9. Ans. (2)

Sol. P(4W4B/2W2B)

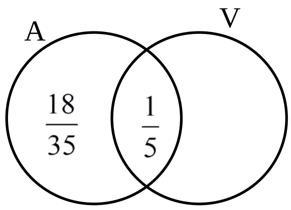
$$=\frac{P(4W4B)\cdot P(2W2B/4W4B)}{P(2W6B)\cdot P(2W2B/2W6B)+P(3W5B)\cdot P(2W2B/3W5B)+\ldots\ldots\ldots\ldots+P(6W2B)\cdot P(2W2B/6W2B)}$$

$$=\frac{\frac{1}{5}\cdot\frac{{}^4C_2\cdot{}^4C_2}{{}^8C_4}}{\frac{1}{5}\cdot\frac{{}^2C_2\cdot{}^6C_2}{{}^8C_4}+\frac{1}{5}\cdot\frac{{}^3C_2\cdot{}^5C_2}{{}^8C_4}+\ldots+\frac{1}{5}\cdot\frac{{}^6C_2\cdot{}^2C_2}{{}^8C_4}}$$

$$=\frac{2}{7}$$

10. Ans. (2)

Sol.



$$P(\bar{A})=\frac{2}{7}=p$$

$$P(A\cap V)=\frac{1}{5}=q$$

$$P(A)=\frac{5}{7}$$

Ans. $P(A\cap\bar{V})=\frac{18}{35}$

11. Ans. (1)

Sol. X Y Z

5 one & 4 five 4 one & 5 five 3 one & 6 five

$$P=\frac{4/9}{5/9+4/9+3/9}=\frac{4}{12}=\frac{1}{3}$$

12. Ans. (3)

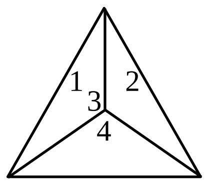
Sol. Favourable cases = ${}^6C_3$

Total out comes = 6^3

Probability of getting greater number than previous one = $\frac{{}^6C_3}{r^3}=\frac{20}{216}=\frac{5}{54}$

13. Ans. (19)

Sol. a, b, c $\in$ {1, 2, 3, 4}



Tetrahedral dice

$ax^2 + bx + c = 0$

has all real roots

$\Rightarrow D \geq 0$

$\Rightarrow b^2 - 4ac \geq 0$

Let b = 1 $\Rightarrow 1 - 4ac \geq 0$ (Not feasible)

b = 2 $\Rightarrow 4 - 4ac \geq 0$

$1 \geq ac \Rightarrow$ a = 1, c = 1,

b = 3 $\Rightarrow 9 - 4ac \geq 0$

$\frac{9}{4} \geq ac$

$\Rightarrow$ a = 1, c = 1

$\Rightarrow$ a = 1, c = 2

$\Rightarrow$ a = 2, c = 1

b = 4 $\Rightarrow 16 - 4ac \geq 0$

$4 \geq ac$

$\Rightarrow$ a = 1, c = 1

$\Rightarrow$ a = 1, c = 2 $\Rightarrow$ a = 2, c = 1

$\Rightarrow$ a = 1, c = 3 $\Rightarrow$ a = 3, c = 1

$\Rightarrow$ a = 1, c = 4 $\Rightarrow$ a = 4, c = 1

$\Rightarrow$ a = 2, c = 2

$$\text{Probability}=\frac{12}{(4)(4)(4)}=\frac{3}{16}=\frac{m}{m}$$

m + n = 19

14. Ans. (2)

Sol.

A	B	C
7R, 5B	5R, 7B	6R, 6B

$P(B) = \frac{1}{3}.\frac{5}{12}+\frac{1}{3}.\frac{7}{12}+\frac{1}{3}.\frac{6}{12}$

required probability $= \dfrac{\frac{1}{3}.\frac{5}{12}}{\frac{1}{3}.\left[\frac{5}{12}+\frac{7}{12}+\frac{6}{12}\right]} = \frac{5}{18}$

15. Ans. (8288)

Sol. $P(W) = \frac{1}{3}$ $\quad P(L) = \frac{2}{3}$

x = number of matches that team wins

y = number of matches that team loses

$|x-y| \le 2$ and $x + y = 10$

$|x-y| = 0, 1, 2 \quad x, y \in N$

Case-I : $|x-y| = 0 \Rightarrow x = y$

$\because x + y = 10 \Rightarrow x = 5 = y$

$P(|x-y|=0) = {}^{10}C_5\left(\frac{1}{3}\right)^5\left(\frac{2}{3}\right)^5$

Case-II : $|x-y| = 1 \Rightarrow x - y = \pm 1$

x = y + 1	x = y – 1
$\because x+y=10$	$\because x+y=10$
2y = 9	2y = 11
Not possible	Not possible

Case-III : $|x-y| = 2 \Rightarrow x - y = \pm 2$

$x - y = 2$ OR $x - y = -2$

$\because x + y = 10$ $\quad \because x + y = 10$

$x = 6, y = 4$ $\quad x = 4, y = 6$

$P(|x-y|=2) = {}^{10}C_6\left(\frac{1}{3}\right)^6\left(\frac{2}{3}\right)^4$

$+{}^{10}C_4\left(\frac{1}{3}\right)^4\left(\frac{2}{3}\right)^6$

$p = {}^{10}C_5\frac{2^5}{3^{10}} + {}^{10}C_6\frac{2^4}{3^{10}} + {}^{10}C_4\frac{2^6}{3^{10}}$

$3^9 p = \frac{1}{3}\left({}^{10}C_5 2^5 + {}^{10}C_6 2^4 + {}^{10}C_4 2^6\right)$

= 8288

16. Ans. (2)

Sol. D > 0

$b^2 > 4ac$

b = 3 : (a, c) = (1, 1)(1, 2)(2,1)

b = 4 : (a, c) = (1, 1)(1, 2)(2,1)(1,3)(3,1)

b = 5 : (a, c) = (1,1)(1,2)(2,1) (1,3)(3,1)(1,4)(4,1) (1,5)(5,1)(1,6)(6,1)(2,3)(3,2)(2,2)

b = 6 : (a, c) = (1,1)(1,2)(2,1) (1,3)(3,1)(1,4)(4,1) (1,5)(5,1)(1,6)(6,1)(2,3)(3,2)(2,4)(4,2)(2,2)

fav. cases = 38

Prob. : $\frac{38}{6\cdot 6\cdot 6}$

17. Ans. (1)

Sol. Total method = 5^3

faverable = ${}^5C_2\left(2^3 - 2\right) = 60$

probability = $\frac{60}{125} = \frac{12}{25}$

18. Ans. (3)

Sol. $ax^2 + bx + c = 0$

a, b, c ∈ {1, 2, 3, 4, 5, 6,7, 8}

Repeated roots D = 0

$\Rightarrow b^2 - 4ac = 0 \Rightarrow b^2 = 4ac$

Prob = $\frac{8}{8\cdot 8\cdot 8} = \frac{1}{64}$

⇒ (a, b, c)

(1, 2, 1) ; (2, 4, 2) ; (1, 4, 4) ; (4, 4, 1) ; (3, 6, 3);

(2, 8, 8) ; (8, 8, 2) ; (4, 8, 4)

8 case

19. Ans. (4)

Sol. x + y = 24, x, y ∈ N

AM > GM ⇒ xy ≤ 144

xy ≥ 108

Favorable pairs of (x, y) are

(13, 11), (12, 12), (14, 10), (15, 9), (16, 8), (17, 7), (18, 6), (6, 18), (7, 17), (8, 16), (9, 15), (10, 14), (11, 13)

i.e. 13 cases

Total choices for x + y = 24 is 23

Probability = $\frac{13}{23} = \frac{m}{n}$

n – m = 10

20. Ans. (1)

Sol.

	A	B
Manufactured	60%	40%
Standard quality	80%	90%

P(Manufactured at B / found standard quality) = ?

A : Found S.Q

B : Manufacture B

C : Manufacture A

$P(E_1) = \frac{40}{100}$

$P(E_2) = \frac{60}{100}$

$P(A/E_1) = \frac{90}{100}$

$P(A/E_2) = \frac{80}{100}$

$\because \quad P(E_1/A) = \frac{P(A/E_1)\,P(E_1)}{P(A/E_1)\,P(E_1) + P(A/E_2)\,P(E_2)}$

$= \frac{3}{7}$

$\therefore$ 126 P = 54

21. Ans. (17)

Sol.

Blue balls	0	1	2	3	4	5
Pr ob.	$\frac{^5C_0\,^4C_1}{^9C_3}$	$\frac{^5C_1\cdot{}^4C_2}{^9C_3}$	$\frac{^5C_2\cdot{}^4C_1}{^9C_3}$	$\frac{^5C_3\cdot{}^4C_0}{^9C_3}$	0	0

$$7\overline{x} = \frac{^5C_1\,^4C_2 + {}^5C_2\cdot{}^4C_1\cdot 2 + {}^5C_3\cdot{}^4C_0\cdot 3}{^9C_3}\cdot 7$$

$$\frac{30+80+30}{84}\cdot 7$$

$$= \frac{140}{12} = \frac{70}{6} = \frac{35}{3}$$

yellow	0	1	2	3	4
		$^5C_2\,^4C_1$	$^5C_1\,^4C_2$	$^5C_0\,^4C_3$	0

$$4\overline{y} = \frac{40+60+12}{84}\cdot 4 = \frac{112}{21} = \frac{16}{3}$$