QUADRATIC EQUATION

1. If α, β are the roots of the equation, $x^2 - x - 1 = 0$ and $S_n = 2023\alpha^n + 2024\beta^n$, then

(1) $2S_{12} = S_{11} + S_{10}$

(2) $S_{12} = S_{11} + S_{10}$

(3) $2S_{11} = S_{12} + S_{10}$

(4) $S_{11} = S_{10} + S_{12}$

2. Let α, β be the roots of the equation $x^2 - x + 2 = 0$ with $\text{Im}(\alpha) > \text{Im}(\beta)$. Then $\alpha^6 + \alpha^4 + \beta^4 - 5\alpha^2$ is equal to

3. Let $\alpha, \beta \in N$ be roots of equation $x^2 - 70x + \lambda = 0$, where $\frac{\lambda}{2}, \frac{\lambda}{3} \notin N$. If λ assumes the minimum possible value, then $\frac{\left(\sqrt{\alpha - 1} + \sqrt{\beta - 1}\right)(\lambda + 35)}{|\alpha - \beta|}$ is equal to :

4. Let S be the set of positive integral values of a for which $\frac{ax^2 + 2(a+1)x + 9a + 4}{x^2 - 8x + 32} < 0, \forall x \in R$.

Then, the number of elements in S is :

(1) 1 (2) 0

(3) ∞ (4) 3

5. Let $\alpha, \beta; \alpha > \beta$, be the roots of the equation $x^2 - \sqrt{2}x - \sqrt{3} = 0$. Let $P_n = \alpha^n - \beta^n$, $n \in N$. Then $\left(11\sqrt{3} - 10\sqrt{2}\right) P_{10} + \left(11\sqrt{2} + 10\right) P_{11} - 11P_{12}$ is equal to :

(1) $10\sqrt{2}P_9$

(2) $10\sqrt{3}P_9$

(3) $11\sqrt{2}P_9$

(4) $11\sqrt{3}P_9$

6. Let α, β be the roots of the equation $x^2 + 2\sqrt{2}x - 1 = 0$. The quadratic equation, whose roots are $\alpha^4 + \beta^4$ and $\frac{1}{10}\left(\alpha^6 + \beta^6\right)$, is :

(1) $x^2 - 190x + 9466 = 0$

(2) $x^2 - 195x + 9466 = 0$

(3) $x^2 - 195x + 9506 = 0$

(4) $x^2 - 180x + 9506 = 0$

7. Let the sum of the maximum and the minimum values of the function $f(x) = \frac{2x^2 - 3x + 8}{2x^2 + 3x + 8}$ be $\frac{m}{n}$, where gcd(m, n) = 1. Then m + n is equal to :

(1) 182 (2) 217

(3) 195 (4) 201

8. If 2 and 6 are the roots of the equation $ax^2 + bx + 1 = 0$, then the quadratic equation, whose roots are $\frac{1}{2a + b}$ and $\frac{1}{6a + b}$, is :

(1) $2x^2 + 11x + 12 = 0$

(2) $4x^2 + 14x + 12 = 0$

(3) $x^2 + 10x + 16 = 0$

(4) $x^2 + 8x + 12 = 0$

9. The number of distinct real roots of the equation $|x|\,|x + 2| - 5|x + 1| - 1 = 0$ is __________.

10. Let α, β be roots of $x^2 + \sqrt{2}x - 8 = 0$. If $U_n = \alpha^n + \beta^n$, then $\frac{U_{10} + \sqrt{12}U_9}{2U_8}$ is equal to _______.

11. Let, α, β be the distinct roots of the equation $x^2 - (t^2 - 5t + 6)x + 1 = 0$, $t \in R$ and $a_n = \alpha^n + \beta^n$. Then the minimum value of $\frac{a_{2023} + a_{2025}}{a_{2024}}$ is

(1) 1/4 (2) –1/2

(3) –1/4 (4) 1/2

SOLUTIONS

1. Ans. (2)

Sol. $x^2 - x - 1 = 0$

$S_n = 2023\alpha^n + 2024\beta^n$

$S_{n-1} + S_{n-2} = 2023\alpha^{n-1} + 2024\beta^{n-1} + 2023\alpha^{n-2} + 2024\beta^{n-2}$

$= 2023\alpha^{n-2}[1+\alpha] + 2024\beta^{n-2}[1+\beta]$

$= 2023\alpha^{n-2}[\alpha^2] + 2024\beta^{n-2}[\beta^2]$

$= 2023\alpha^n + 2024\beta^n$

$S_{n-1} + S_{n-2} = S_n$

Put n = 12

$S_{11} + S_{10} = S_{12}$

2. Ans. (13)

Sol. $\alpha^6 + \alpha^4 + \beta^4 - 5\alpha^2$

$= \alpha^4(\alpha - 2) + \alpha^4 - 5\alpha^2 + (\beta - 2)^2$

$= \alpha^5 - \alpha^4 - 5\alpha^2 + \beta^2 - 4\beta + 4$

$= \alpha^3(\alpha - 2) - \alpha^4 - 5\alpha^2 + \beta - 2 - 4\beta + 4$

$= -2\alpha^3 - 5\alpha^2 - 3\beta + 2$

$= -2\alpha(\alpha - 2) - 5\alpha^2 - 3\beta + 2$

$= -7\alpha^2 + 4\alpha - 3\beta + 2$

$= -7(\alpha - 2) + 4\alpha - 3\beta + 2$

$= -3\alpha - 3\beta + 16 = -3(1) + 16 = 13$

3. Ans. (60)

Sol. $x^2 - 70x + \lambda = 0$

$\alpha + \beta = 70$

$\alpha\beta = \lambda$

$\therefore \alpha(70 - \alpha) = \lambda$

Since, 2 and 3 does not divide λ

$\therefore \alpha = 5, \beta = 65, \lambda = 325$

By putting value of α, β, λ we get the required value 60.

4. Ans. (2)

Sol. $ax^2 + 2(a + 1)x + 9a + 4 < 0 \ \forall x \in R$

$\therefore a < 0$

5. Ans. (2)

Sol. $x^2 - \sqrt{2}x - \sqrt{3} = 0 \left\langle \begin{matrix}\alpha \\ \beta\end{matrix}\right.$

$\alpha^{n+2} - \sqrt{2}\alpha^{n+1} - \sqrt{3}\alpha^n = 0$

and $\beta^{n+2} - \sqrt{2}\beta^{n+1} - \sqrt{3}\beta^n = 0$

Subtracting

$(\alpha^{n+2} - \beta^{n+2}) - \sqrt{2}(\alpha^{n+1} - \beta^{n+1}) - \sqrt{3}(\alpha^n - \beta^n) = 0$

$\Rightarrow P_{n+2} - \sqrt{2}P_{n+1} - \sqrt{3}P_n = 0$

Put n = 10

$P_{12} - \sqrt{2}P_{11} - \sqrt{3}P_{10} = 0$

n = 9

$P_{11} - \sqrt{2}P_{10} - \sqrt{3}P_9 = 0$

$11\left(\sqrt{3}.P_{10} + \sqrt{2}P_{11} - P_{11}\right) - 10\left(\sqrt{2}P_{10} - P_{11}\right)$

$= 0 - 10\left(-\sqrt{3}P_9\right) = 10\sqrt{3}P_9$

6. Ans. (3)

Sol. $x^2 + 2\sqrt{2}\,x - 1 = 0$

$\alpha + \beta = -2\sqrt{2}$

$\alpha\beta = -1$

$\alpha^4 + \beta^4 = (\alpha^2 + \beta^2)^2 - 2\alpha^2\beta^2$

$= ((\alpha + \beta)^2 - 2\alpha\beta)^2 - 2(\alpha\beta)^2$

$= (8 + 2)^2 - 2(-1)^2$

$= 100 - 2 = 98$

$\alpha^6 + \beta^6 = (\alpha^3 + \beta^3)^2 - 2\alpha^3\beta^3$

$= ((\alpha + \beta)((\alpha + \beta)^2 - 3\alpha\beta))^2 - 2(\alpha\beta)^3$

$= (-2\sqrt{2}(8 + 3))^2 + 2$

$= (8)(121) + 2 = 970$

$\frac{1}{10}(\alpha^6 + \beta^6) = 97$

$x^2 - (98 + 97)x + (98)(97) = 0$

$\Rightarrow x^2 - 195x + 9506 = 0$

7. Ans. (4)

Sol. $y = \dfrac{2x^2 - 3x + 8}{2x^2 + 3x + 8}$

$x^2(2y-2) + x(3y+3) + 8y - 8 = 0$

use $D \geq 0$

$(3y+3)^2 - 4(2y-2)(8y-8) \geq 0$

$(11y-5)(5y-11) \leq 0$

$\Rightarrow y \in \left[\dfrac{5}{11}, \dfrac{11}{5}\right]$

y = 1 is also included

8. Ans. (4)

Sol. Sum $= 8 = -\dfrac{b}{a}$

Product $= 12 = \dfrac{1}{a}$ $\Rightarrow a = \dfrac{1}{12}$

$b = -\dfrac{2}{3}$

$2a + b = \dfrac{2}{12} - \dfrac{2}{3} = -\dfrac{1}{2}$

$6a + b = \dfrac{6}{12} - \dfrac{2}{3} = -\dfrac{1}{6}$

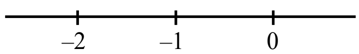
sum $= -8$

P = 12

$x^2 + 8x + 12 = 0$

9. Ans. (3)

Sol.



Case-1

$x \geq 0$

$x^2 + 2x - 5x - 5 - 1 = 0$

$x^2 - 3x - 6 = 0$

$x = \dfrac{3 \pm \sqrt{9+24}}{2} = \dfrac{3 \pm \sqrt{33}}{2}$

One positive root

Case-2

$-1 \leq x < 0$

$-x^2 - 2x - 5x - 5 - 1 = 0$

$x^2 + 7x + 6 = 0$

$(x+6)(x+1) = 0$

$x = -1$

one root in range

Case-3

$-2 \leq x < -1$

$x^2 - 2x + 5x + 5 - 1 = 0$

$x^2 - 3x - 4 = 0$

$(x-4)(x+1) = 0$

No root in range

Case-4

$x < -2$

$x^2 + 7x + 4 = 0$

$x = \dfrac{-7 \pm \sqrt{49-16}}{2} = \dfrac{7 \pm \sqrt{33}}{2}$

one root in range

Total number of distinct roots are 3

10. Ans. (4)

Sol. $\dfrac{\alpha^{10} + \beta^{10} + \sqrt{2}\left(\alpha^9 + \beta^9\right)}{2\left(\alpha^8 + \beta^8\right)}$

$\dfrac{\alpha^8\left(\alpha^2 + \sqrt{2}\alpha\right) + \beta^8\left(\beta^2 + \sqrt{2}\beta\right)}{2\left(\alpha^8 + \beta^8\right)}$

$\dfrac{8\alpha^8 + 8\beta^8}{2\left(\alpha^8 + \beta^8\right)} = 4$

11. Ans. (3)

Sol. by newton's theorem

$a_{n+2} - (t^2 - 5t + 6)a_{n+1} + a_n = 0$

$\therefore\ a_{2025} + a_{2023} = (t^2 - 5t + 6)\, a_{2024}$

$\therefore\ \dfrac{a_{2025} + a_{2023}}{a_{2024}} = t^2 - 5t + 6$

$\because\ t^2 - 5t + 6 = \left(t - \dfrac{5}{2}\right)^2 - \dfrac{1}{4}$

$\therefore$ minimum value $= -\dfrac{1}{4}$