

SEQUENCE & SERIES

1. The number of common terms in the progressions 4, 9, 14, 19,, up to 25th term and 3, 6, 9, 12,, up to 37th term is :
 (1) 9 (2) 5
 (3) 7 (4) 8
2. If $8 = 3 + \frac{1}{4}(3+p) + \frac{1}{4^2}(3+2p) + \frac{1}{4^3}(3+3p) + \dots$, then the value of p is _____.
3. The 20th term from the end of the progression $20, 19\frac{1}{4}, 18\frac{1}{2}, 17\frac{3}{4}, \dots, -129\frac{1}{4}$ is :-
 (1) -118 (2) -110
 (3) -115 (4) -100
4. If in a G.P. of 64 terms, the sum of all the terms is 7 times the sum of the odd terms of the G.P, then the common ratio of the G.P. is equal to
 (1) 7 (2) 4
 (3) 5 (4) 6
5. In an A.P., the sixth term $a_6 = 2$. If the a_1, a_4, a_5 is the greatest, then the common difference of the A.P., is equal to
 (1) $\frac{3}{2}$ (2) $\frac{8}{5}$ (3) $\frac{2}{3}$ (4) $\frac{5}{8}$
6. If $\log_e a, \log_e b, \log_e c$ are in an A.P. and $\log_e a - \log_e 2b, \log_e 2b - \log_e 3c, \log_e 3c - \log_e a$ are also in an A.P, then $a : b : c$ is equal to
 (1) 9 : 6 : 4 (2) 16 : 4 : 1
 (3) 25 : 10 : 4 (4) 6 : 3 : 2
7. If each term of a geometric progression a_1, a_2, a_3, \dots with $a_1 = \frac{1}{8}$ and $a_2 \neq a_1$, is the arithmetic mean of the next two terms and $S_n = a_1 + a_2 + \dots + a_n$, then $S_{20} - S_{18}$ is equal to
 (1) 2^{15} (2) -2^{18}
 (3) 2^{18} (4) -2^{15}
8. Let S_n denote the sum of first n terms an arithmetic progression. If $S_{20} = 790$ and $S_{10} = 145$, then $S_{15} - S_5$ is :
 (1) 395 (2) 390
 (3) 405 (4) 410

9. Let $\alpha = 1^2 + 4^2 + 8^2 + 13^2 + 19^2 + 26^2 + \dots$ upto 10 terms and $\beta = \sum_{n=1}^{10} n^4$. If $4\alpha - \beta = 55k + 40$, then k is equal to _____.
10. Let a and b be two distinct positive real numbers. Let 11th term of a GP, whose first term is a and third term is b, is equal to pth term of another GP, whose first term is a and fifth term is b. Then p is equal to
 (1) 20 (2) 25
 (3) 21 (4) 24
11. Let S_n be the sum to n-terms of an arithmetic progression 3, 7, 11,, .
 If $40 < \left(\frac{6}{n(n+1)} \sum_{k=1}^n S_k \right) < 42$, then n equals _____.
12. For $0 < c < b < a$, let
 $(a + b - 2c)x^2 + (b + c - 2a)x + (c + a - 2b) = 0$
 and $\alpha \neq 1$ be one of its root. Then, among the two statements
 (I) If $\alpha \in (-1, 0)$, then b cannot be the geometric mean of a and c
 (II) If $\alpha \in (0, 1)$, then b may be the geometric mean of a and c
 (1) Both (I) and (II) are true
 (2) Neither (I) nor (II) is true
 (3) Only (II) is true
 (4) Only (I) is true
13. The sum of the series $\frac{1}{1 - 3 \cdot 1^2 + 1^4} + \frac{2}{1 - 3 \cdot 2^2 + 2^4} + \frac{3}{1 - 3 \cdot 3^2 + 3^4} + \dots$ up to 10 terms is
 (1) $\frac{45}{109}$ (2) $-\frac{45}{109}$
 (3) $\frac{55}{109}$ (4) $-\frac{55}{109}$

26. The value of $\frac{1 \cdot 2^2 + 2 \cdot 3^2 + \dots + 100 \cdot (101)^2}{1^2 \cdot 2 + 2^2 \cdot 3 + \dots + 100^2 \cdot 101}$ is

- (1) $\frac{306}{305}$ (2) $\frac{305}{301}$
 (3) $\frac{32}{31}$ (4) $\frac{31}{30}$

27. For $x \geq 0$, the least value of K, for which $4^{1+x} + 4^{1-x}, \frac{K}{2}, 16^x + 16^{-x}$ are three consecutive terms of an A.P. is equal to :

- (1) 10 (2) 4
 (3) 8 (4) 16

28. Let ABC be an equilateral triangle. A new triangle is formed by joining the middle points of all sides of the triangle ABC and the same process is repeated infinitely many times. If P is the sum of perimeters and Q is be the sum of areas of all the triangles formed in this process, then:

- (1) $P^2 = 36\sqrt{3}Q$ (2) $P^2 = 6\sqrt{3}Q$
 (3) $P = 36\sqrt{3}Q^2$ (4) $P^2 = 72\sqrt{3}Q$

29. A software company sets up m number of computer systems to finish an assignment in 17 days. If 4 computer systems crashed on the start of the second day, 4 more computer systems crashed on the start of the third day and so on, then it took 8 more days to finish the assignment. The value of m is equal to :

- (1) 125 (2) 150
 (3) 180 (4) 160

30. If $S(x) = (1 + x) + 2(1 + x)^2 + 3(1 + x)^3 + \dots + 60(1 + x)^{60}$, $x \neq 0$, and $(60)^2 S(60) = a(b)^b + b$, where $a, b \in \mathbb{N}$, then $(a + b)$ equal to _____.

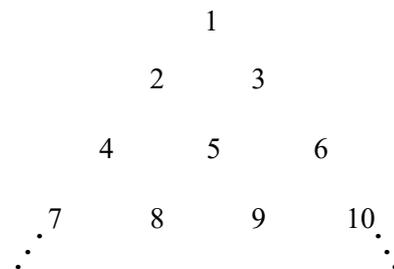
31. Let a_1, a_2, a_3, \dots be in an arithmetic progression of positive terms.

Let $A_k = a_1^2 - a_2^2 + a_3^2 - a_4^2 + \dots + a_{2k-1}^2 - a_{2k}^2$.

If $A_3 = -153, A_5 = -435$ and $a_1^2 + a_2^2 + a_3^2 = 66$, then $a_{17} - A_7$ is equal to _____.

32. Let the first term of a series be $T_1 = 6$ and its r^{th} term $T_r = 3 T_{r-1} + 6^r, r = 2, 3, \dots, n$. If the sum of the first n terms of this series is $\frac{1}{5}(n^2 - 12n + 39)(4 \cdot 6^n - 5 \cdot 3^n + 1)$. Then n is equal to _____.

33. Let the positive integers be written in the form :



If the k^{th} row contains exactly k numbers for every natural number k, then the row in which the number 5310 will be, is _____.

SOLUTIONS

1. **Ans. (3)**

Sol. 4, 9, 14, 19, ..., up to 25th term

$$T_{25} = 4 + (25 - 1)5 = 4 + 120 = 124$$

3, 6, 9, 12, ..., up to 37th term

$$T_{37} = 3 + (37 - 1)3 = 3 + 108 = 111$$

Common difference of Ist series $d_1 = 5$

Common difference of IInd series $d_2 = 3$

First common term = 9, and

their common difference = 15 (LCM of d_1 and d_2)

then common terms are

9, 24, 39, 54, 69, 84, 99

2. **Ans. (9)**

$$\text{Sol. } 8 = \frac{3}{1 - \frac{1}{4}} + \frac{p \cdot \frac{1}{4}}{\left(1 - \frac{1}{4}\right)^2}$$

(sum of infinite terms of A.G.P = $\frac{a}{1-r} + \frac{dr}{(1-r)^2}$)

$$\Rightarrow \frac{4p}{9} = 4 \Rightarrow p = 9$$

3. **Ans. (3)**

Sol. 20, $19\frac{1}{4}$, $18\frac{1}{2}$, $17\frac{3}{4}$,, $-129\frac{1}{4}$

This is A.P. with common difference

$$d_1 = -1 + \frac{1}{4} = -\frac{3}{4}$$

$-129\frac{1}{4}, \dots, 19\frac{1}{4}, 20$

This is also A.P. $a = -129\frac{1}{4}$ and $d = \frac{3}{4}$

$$\text{Required term} = -129\frac{1}{4} + (20 - 1)\left(\frac{3}{4}\right)$$

$$= -129 - \frac{1}{4} + 15 - \frac{3}{4} = -115$$

4. **Ans. (4)**

$$\begin{aligned} \text{Sol. } & a + ar + ar^2 + ar^3 + \dots + ar^{63} \\ & = 7(a + ar^2 + ar^4 + \dots + ar^{62}) \\ & \Rightarrow \frac{a(1-r^{64})}{1-r} = \frac{7a(1-r^{64})}{1-r^2} \\ & \qquad \qquad \qquad r = 6 \end{aligned}$$

5. **Ans. (2)**

$$\text{Sol. } a_6 = 2 \Rightarrow a + 5d = 2$$

$$a_1 a_5 = a(a + 3d)(a + 4d)$$

$$= (2 - 5d)(2 - 2d)(2 - d)$$

$$f(d) = 8 - 32d + 34d^2 - 20d + 30d^2 - 10d^3$$

$$f'(d) = -2(5d - 8)(3d - 2)$$

$$\frac{-}{2/3} \quad \frac{+}{8/5} \quad \frac{-}{}$$

$$d = \frac{8}{5}$$

6. **Ans. (1)**

Sol. $\log_e a, \log_e b, \log_e c$ are in A.P.

$$\therefore b^2 = ac \dots (i)$$

Also

$\log_e \left(\frac{a}{2b}\right), \log_e \left(\frac{2b}{3c}\right), \log_e \left(\frac{3c}{a}\right)$ are in A.P.

$$\left(\frac{2b}{3c}\right)^2 = \frac{a}{2b} \cdot \frac{3c}{a}$$

$$\frac{b}{c} = \frac{3}{2}$$

Putting in eq. (i) $b^2 = a \times \frac{2b}{3}$

$$\frac{a}{b} = \frac{3}{2}$$

$$a : b : c = 9 : 6 : 4$$

7. **Ans. (4)**

Sol. Let rth term of the GP be ar^{r-1} . Given,

$$2a_r = a_{r+1} + a_{r+2}$$

$$2ar^{r-1} = ar^n + ar^{n+1}$$

$$\frac{2}{r} = 1 + r$$

$$r^2 + r - 2 = 0$$

Hence, we get, $r = -2$ (as $r \neq 1$)

So, $S_{20} - S_{18} = (\text{Sum upto 20 terms}) - (\text{Sum upto 18 terms}) = T_{19} + T_{20}$

$$T_{19} + T_{20} = ar^{18} (1 + r)$$

Putting the values $a = \frac{1}{8}$ and $r = -2$;

$$\text{we get } T_{19} + T_{20} = -2^{15}$$

8. Ans. (1)

Sol. $S_{20} = \frac{20}{2}[2a + 19d] = 790$

$$2a + 19d = 79 \quad \dots(1)$$

$$S_{10} = \frac{10}{2}[2a + 9d] = 145$$

$$2a + 9d = 29 \quad \dots(2)$$

From (1) and (2) $a = -8, d = 5$

$$\begin{aligned} S_{15} - S_5 &= \frac{15}{2}[2a + 14d] - \frac{5}{2}[2a + 4d] \\ &= \frac{15}{2}[-16 + 70] - \frac{5}{2}[-16 + 20] \\ &= 405 - 10 = 395 \end{aligned}$$

9. Ans. (353)

Sol. $\alpha = 1^2 + 4^2 + 8^2 \dots$

$$t_n = an^2 + bn + c$$

$$1 = a + b + c$$

$$4 = 4a + 2b + c$$

$$8 = 9a + 3b + c$$

On solving we get, $a = \frac{1}{2}, b = \frac{3}{2}, c = -1$

$$\alpha = \sum_{n=1}^{10} \left(\frac{n^2}{2} + \frac{3n}{2} - 1 \right)^2$$

$$4\alpha = \sum_{n=1}^{10} (n^2 + 3n - 2)^2, \beta = \sum_{n=1}^{10} n^4$$

$$4\alpha - \beta = \sum_{n=1}^{10} (6n^3 + 5n^2 - 12n + 4) = 55(353) + 40$$

10. Ans. (3)

Sol. 1st GP $\Rightarrow t_1 = a, t_3 = b = ar^2 \Rightarrow r^2 = \frac{b}{a}$

$$t_{11} = ar^{10} = a(r^2)^5 = a \cdot \left(\frac{b}{a} \right)^5$$

$$2^{\text{nd}} \text{ G.P. } \Rightarrow T_1 = a, T_5 = ar^4 = b$$

$$\Rightarrow r^4 = \left(\frac{b}{a} \right) \Rightarrow \left(\frac{b}{a} \right)^{1/4}$$

$$T_p = ar^{p-1} = a \left(\frac{b}{a} \right)^{\frac{p-1}{4}}$$

$$t_{11} = T_p \Rightarrow a \left(\frac{b}{a} \right)^5 = a \left(\frac{b}{a} \right)^{\frac{p-1}{4}}$$

$$\Rightarrow 5 = \frac{p-1}{4} \Rightarrow p = 21$$

11. Ans. (9)

Sol. $S_n = 3 + 7 + 11 + \dots n$ terms

$$= \frac{n}{2}(6 + (n-1)4) = 3n + 2n^2 - 2n$$

$$= 2n^2 + n$$

$$\sum_{k=1}^n S_k = 2 \sum_{k=1}^n K^2 + \sum_{k=1}^n K$$

$$= 2 \cdot \frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}$$

$$= n(n+1) \left[\frac{2n+1}{3} + \frac{1}{2} \right]$$

$$= \frac{n(n+1)(4n+5)}{6}$$

$$\Rightarrow 40 < \frac{6}{n(n+1)} \sum_{k=1}^n S_k < 42$$

$$40 < 4n + 5 < 42$$

$$35 < 4n < 37$$

$$n = 9$$

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12. Ans. (1)

Sol. $f(x) = (a + b - 2c)x^2 + (b + c - 2a)x + (c + a - 2b)$

$$f(1) = a + b - 2c + b + c - 2a + c + a - 2b = 0$$

$$f(1) = 0$$

$$\therefore \alpha \cdot 1 = \frac{c + a - 2b}{a + b - 2c}$$

$$\alpha = \frac{c + a - 2b}{a + b - 2c}$$

$$\text{If, } -1 < \alpha < 0$$

$$-1 < \frac{c + a - 2b}{a + b - 2c} < 0$$

$$b + c < 2a \text{ and } b > \frac{a + c}{2}$$

therefore, b cannot be G.M. between a and c.

$$\text{If, } 0 < \alpha < 1$$

$$0 < \frac{c + a - 2b}{a + b - 2c} < 1$$

$$b > c \text{ and } b < \frac{a + c}{2}$$

Therefore, b may be the G.M. between a and c.

13. Ans. (4)

Sol. General term of the sequence,

$$T_r = \frac{r}{1 - 3r^2 + r^4}$$

$$T_r = \frac{r}{r^4 - 2r^2 + 1 - r^2}$$

$$T_r = \frac{r}{(r^2 - 1)^2 - r^2}$$

$$T_r = \frac{r}{(r^2 - r - 1)(r^2 + r - 1)}$$

$$T_r = \frac{1}{2} \left[\frac{(r^2 + r - 1) - (r^2 - r - 1)}{(r^2 - r - 1)(r^2 + r - 1)} \right]$$

$$= \frac{1}{2} \left[\frac{1}{r^2 - r - 1} - \frac{1}{r^2 + r - 1} \right]$$

Sum of 10 terms,

$$\sum_{r=1}^{10} T_r = \frac{1}{2} \left[\frac{1}{-1} - \frac{1}{109} \right] = \frac{-55}{109}$$

14. Ans. (4)

Sol. $1 + d, 1 + 7d, 1 + 43d$ are in GP

$$(1 + 7d)^2 = (1 + d)(1 + 43d)$$

$$1 + 49d^2 + 14d = 1 + 44d + 43d^2$$

$$6d^2 - 30d = 0$$

$$d = 5$$

$$S_{20} = \frac{20}{2} [2 \cdot 1 + (20 - 1) \cdot 5]$$

$$= 10 [2 + 95]$$

$$= 970$$

15. Ans. (4)

Sol. $3, a, b, c \rightarrow$ A.P $\Rightarrow 3, 3+d, 3+2d, 3+3d$

$3, a-1, b+1, c+9 \rightarrow$ G.P $\Rightarrow 3, 2+d, 4+2d, 12+3d$

$$a = 3 + d \qquad (2 + d)^2 = 3(4 + 2d)$$

$$b = 3 + 2d \qquad d = 4, -2$$

$$c = 3 + 3d$$

If $d = 4$ G.P. $\Rightarrow 3, 6, 12, 24$

$$b = 11$$

$$c = 15$$

$$\frac{a + b + c}{3} = 11$$

16. Ans. (6699)

Sol. $3, 7, 11, 15, \dots, 403$

$2, 5, 8, 11, \dots, 404$

LCM (4, 3) = 12

$11, 23, 35, \dots$ let (403)

$$403 = 11 + (n - 1) \times 12$$

$$\frac{392}{12} = n - 1$$

$$33.66 = n$$

$$n = 33$$

$$\text{Sum} = \frac{33}{2} (22 + 32 \cdot 12)$$

$$= 6699$$

17. Ans. (1)

Sol. $px^2 + qx - r = 0 \begin{matrix} \alpha \\ \beta \end{matrix}$

$p = A, q = AR, r = AR^2$
 $Ax^2 + ARx - AR^2 = 0$

$x^2 + Rx - R^2 = 0 \begin{matrix} \alpha \\ \beta \end{matrix}$

$\therefore \frac{1}{\alpha} + \frac{1}{\beta} = \frac{3}{4}$

$\therefore \frac{\alpha + \beta}{\alpha\beta} = \frac{3}{4}$

$\Rightarrow \frac{-R}{-R^2} = \frac{3}{4} \Rightarrow R = \frac{4}{3}$

$(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta = R^2 - 4(-R^2) = 5\left(\frac{16}{9}\right)$
 $= \frac{80}{9}$

18. Ans. (3)

Sol. $S_{10} = 390$

$\frac{10}{2}[2a + (10-1)d] = 390$

$\Rightarrow 2a + 9d = 78 \dots\dots(1)$

$\frac{t_{10}}{t_5} = \frac{15}{7} \Rightarrow \frac{a+9d}{a+4d} = \frac{15}{7} \Rightarrow 8a = 3d \dots\dots(2)$

From (1) & (2)

$a = 3 \text{ \& } d = 8$

$S_{15} - S_5 = \frac{15}{2}(6 + 14 \cdot 8) - \frac{5}{2}(6 + 4 \cdot 8)$
 $= \frac{15 \cdot 118 - 5 \cdot 38}{2} = 790$

19. Ans. (3)

Sol. $T_2 + T_6 = \frac{70}{3}$

$ar + ar^5 = \frac{70}{3}$

$T_3 \cdot T_5 = 49$

$ar^2 \cdot ar^4 = 49$

$a^2r^6 = 49$

$ar^3 = +7, a = \frac{7}{r^3}$

$ar(1 + r^4) = \frac{70}{3}$

$\frac{7}{r^2}(1 + r^4) = \frac{70}{3}, r^2 = t$

$\frac{1}{t}(1 + t^2) = \frac{10}{3}$

$3t^2 - 10t + 3 = 0$

$t = 3, \frac{1}{3}$

Increasing G.P. $r^2 = 3, r = \sqrt{3}$

$T_4 + T_6 + T_8$

$= ar^3 + ar^5 + ar^7$

$= ar^3(1 + r^2 + r^4)$

$= 7(1 + 3 + 9) = 91$

20. Ans. (1505)

Sol. 2, 5, 11, 20,

General term = $\frac{3n^2 - 3n + 4}{2}$

$T_{10} = \frac{3(100) - 3(10) + 4}{2}$

$= 137$

10 terms with c.d. = 3

sum = $\frac{10}{2}(2(137) + 9(3))$

$= 1505$

21. Ans. (4)

Sol. $\sum_{n=0}^{\infty} ar^n = 57$

$a + ar + ar^2 + \dots = 57$

$\frac{a}{1-r} = 57 \dots\dots(I)$

$\sum_{n=0}^{\infty} a^3 r^{3n} = 9747$

$a^3 + a^3 \cdot r^3 + a^3 \cdot r^6 + \dots = 9746$

$\frac{a^3}{1-r^3} = 9746 \dots\dots(II)$

$$\frac{(I)^3}{(II)} \Rightarrow \frac{\frac{a^3}{(1-r)^3}}{\frac{a^3}{1-r^3}} = \frac{57^3}{9717} = 19$$

On solving, $r = \frac{2}{3}$ and $r = \frac{3}{2}$ (rejected)

$$a = 19$$

$$\therefore a + 18r = 19 + 18 \cdot \frac{2}{3} = 31$$

22. **Ans. (1011)**

Sol. $\left(\frac{1}{\alpha+1} + \frac{1}{\alpha+2} + \dots + \frac{1}{\alpha+2012} \right)$
 $-\left\{ \left(\frac{1}{1} - \frac{1}{2} \right) + \left(\frac{1}{3} - \frac{1}{4} \right) + \dots + \left(\frac{1}{2023} - \frac{1}{2024} \right) \right\} = \frac{1}{2024}$
 $\Rightarrow \left(\frac{1}{\alpha+1} + \frac{1}{\alpha+2} + \dots + \frac{1}{\alpha+2012} \right)$
 $-\left\{ \left(\frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} \right) + \dots + \frac{1}{2023} \right.$
 $\left. - \frac{1}{2024} - 2 \left(\frac{1}{2} + \frac{1}{4} + \dots + \frac{1}{2022} \right) \right\} = \frac{1}{2024}$
 $\Rightarrow \left(\frac{1}{\alpha+1} + \frac{1}{\alpha+2} + \dots + \frac{1}{\alpha+2012} \right)$
 $-\left(\frac{1}{1} + \frac{1}{2} + \dots + \frac{1}{2023} \right)$
 $+ \frac{1}{2024} + \left(\frac{1}{1} + \frac{1}{2} + \dots + \frac{1}{1011} \right) = \frac{1}{2024}$
 $\Rightarrow \frac{1}{\alpha+1} + \frac{1}{\alpha+2} + \dots + \frac{1}{\alpha+2012}$
 $= \frac{1}{1012} + \frac{1}{1013} + \dots + \frac{1}{2023}$
 $\Rightarrow \alpha = 1011$

23. **Ans. (2)**

Sol. $\frac{1}{1 \cdot (1+d)} + \frac{1}{(1+d)(1+2d)} + \dots$
 $\frac{1}{(1+9d)(1+10d)} = 5$
 $\frac{1}{d} \left[\frac{(1+d) - 1}{1 \cdot (1+d)} + \frac{(1+2d) - (1-d)}{(1+d)(1+2d)} \right] + \dots$
 $\frac{(1+10d) - (1+9d)}{(1+9d)(1+10d)} = 5$

$$\frac{1}{d} \left[\left(1 - \frac{1}{1+d} \right) + \left(\frac{1}{1+d} - \frac{1}{1+2d} \right) + \dots \right]$$

$$\left(\frac{1}{1+9d} - \frac{1}{1+10d} \right) = 5$$

$$\frac{1}{d} \left[1 - \frac{1}{(1+10d)} \right] = 5$$

$$\frac{10d}{1+10d} = 5d$$

$$50d = 5$$

24. **Ans. (4)**

Sol. $p^2 = 2q$

$$2 = a + 6d \quad \dots(i)$$

$$p = a + 7d \quad \dots(ii)$$

$$q = a + 12d \quad \dots(iii)$$

$$p - 2 = d \quad \dots(ii) - (i)$$

$$q - p = 5d \quad \dots(iii) - (ii)$$

$$q - p = 5(p - 2)$$

$$q = 6p - 10$$

$$p^2 = 2(6p - 10)$$

$$p^2 - 12p + 20 = 0$$

$$p = 10, 2$$

$$p = 10 ; q = 50$$

$$d = 8$$

$$a = -46$$

$$2, 10, 50, 250, 1250$$

$$ar^4 = a + (n - 1)d$$

$$1250 = -46 + (n - 1)8$$

$$n = 163$$

25. **Ans. (1)**

Sol. $2b = a + c, b^2 = (a + 1)(c + 3),$

$$\frac{a + b + c}{3} = 8 \rightarrow \underline{b} = 8, a + c = 16$$

$$64 = (a + 1)(19 - a) = 19 + 18a - a^2$$

$$a^2 - 18a - 45 = 0 \rightarrow (a - 15)(a + 3) = 0, (a > 10)$$

$$a = 15, c = 1, b = 8$$

$$((abc)^{1/3})^3 = abc = 120$$

26. Ans. (2)

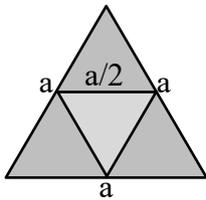
$$\begin{aligned} \text{Sol. } \frac{1 \cdot 2^2 + 2 \cdot 3^2 + \dots + 100 \cdot (101)^2}{1^2 \cdot 2 + 2^2 \cdot 3 + \dots + 100^2 \cdot 101} &= \frac{\sum_{r=1}^{100} r(r+1)^2}{\sum_{r=1}^{100} r^2(r+1)} \\ &= \frac{\sum_{r=1}^{100} (r^3 + 2r^2 + r)}{\sum_{r=1}^{100} (r^3 + r^2)} = \frac{\left[\frac{n(n+1)^2}{2} \right] + \frac{2 \cdot n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}}{\left[\frac{n(n+1)^2}{2} \right] + \frac{n(n+1)(2n+1)}{6}} \\ &= \frac{\frac{n(n+1)}{2} \left[\frac{n(n+1)}{2} + \frac{2}{3} \cdot (2n+1) + 1 \right]}{\frac{n(n+1)}{2} \left[\frac{n(n+1)}{2} + \frac{(2n+1)}{3} \right]} ; \text{ Put } n = 100 \\ &= \frac{\frac{100(101)}{2} + \frac{2}{3}(201) + 1}{\frac{100 \cdot 101}{2} + \frac{201}{3}} = \frac{5185}{5117} = \frac{305}{301} \end{aligned}$$

27. Ans. (1)

$$\begin{aligned} \text{Sol. } k &= 4 \left(4^x + \frac{1}{4^x} \right) + \left(4^{2x} + \frac{1}{4^{2x}} \right) \\ &\geq 2 \qquad \qquad \qquad \geq 2 \\ k &\geq 10 \end{aligned}$$

28. Ans. (1)

Sol.



$$\text{Area of first } \Delta = \frac{\sqrt{3}a^2}{4}$$

$$\text{Area of second } \Delta = \frac{\sqrt{3}a^2}{4} \cdot \frac{a^2}{4} = \frac{\sqrt{3}a^2}{16}$$

$$\text{Area of third } \Delta = \frac{\sqrt{3}a^2}{64}$$

$$\text{sum of area} = \frac{\sqrt{3}a^2}{4} \left(1 + \frac{1}{4} + \frac{1}{16} \dots \right)$$

$$Q = \frac{\sqrt{3}a^2}{4} \cdot \frac{1}{\frac{3}{4}} = \frac{a^2}{\sqrt{3}}$$

$$\text{perimeter of 1st } \Delta = 3a$$

$$\text{perimeter of 2nd } \Delta = \frac{3a}{2}$$

$$\text{perimeter of 3rd } \Delta = \frac{3a}{4}$$

$$P = 3a \left(1 + \frac{1}{2} + \frac{1}{4} + \dots \right)$$

$$P = 3a \cdot 2 = 6a$$

$$a = \frac{P}{6}$$

$$Q = \frac{1}{\sqrt{3}} \cdot \frac{P^2}{36}$$

$$P^2 = 36\sqrt{3}Q$$

29. Ans. (2)

$$\text{Sol. } 17m = m + (m-4) + (m-4 \times 2) + \dots + (m-4 \times 24)$$

$$17m = 25m - 4(1 + 2 + \dots + 24)$$

$$8m = \frac{4 \cdot 24 \cdot 25}{2} = 150$$

30. Ans. (3660)

$$\text{Sol. } S(x) = (1+x) + 2(1+x^2) + 3(1+x^3) + \dots + 60(1+x^{60})$$

$$(1+x)S = (1+x)^2 + \dots + 59(1+x)^{60} + 60(1+x)^{61}$$

$$-xS = \frac{(1+x)(1+x)^{60} - 1}{x} - 60(1+x)^{61}$$

$$\text{Put } x = 60$$

$$-60S = \frac{61((61)^{60} - 1)}{60} - 60(61)^{61}$$

on solving 3660

31. Ans. (910)

Sol. $d \rightarrow$ common diff.

$$A_k = -kd[2a + (2k-1)d]$$

$$A_3 = -153$$

$$\Rightarrow 153 = 13d[2a + 5d]$$

$$51 = d[2a + 5d] \quad \dots(1)$$

$$A_5 = -435$$

$$435 = 5d[2a + 9d]$$

$$87 = d[2a + 9d]$$

$$(2) - (1)$$

$$36 = 4d^2$$

$$d = 3, a = 1$$

$$a_{17} - A_7 = 49 - [-7.3[2 + 39]] = 910$$

SEQUENCE & SERIES

32. Ans. (6)

Sol. $T_r = 3T_{r-1} + 6^r, r = 2, 3, 4, \dots, n$

$$T_2 = 3.T_1 + 6^2$$

$$T_2 = 3.6 + 6^2 \quad \dots(1)$$

$$T_3 = 3T_2 + 6^3$$

$$T_3 = 3T_2 + 6^3$$

$$T_3 = 3(3.6 + 6^2) + 6^3$$

$$T_3 = 3^2.6 + 3.6^2 + 6^3 \quad \dots(2)$$

$$T_r = 3^{r-1}.6 + 3^{r-2}.6^2 + \dots + 6^r$$

$$T_r = 3^{r-1} \cdot 6 \left[1 + \frac{6}{3} + \left(\frac{6}{3}\right)^2 + \dots + \left(\frac{6}{3}\right)^{r-1} \right]$$

$$T_r = 3^{r-1}.6(1 + 2 + 2^2 + \dots + 2^{r-1})$$

$$T_r = 6 \cdot 3^{r-1} \cdot \frac{(1-2^r)}{(-1)}$$

$$T_r = 6.3^{r-1} \cdot (2^r - 1)$$

$$T_r = \frac{6 \cdot 3^r}{3} \cdot (2^r - 1)$$

$$T_r = 2 \cdot (6^r - 3^r)$$

$$S_n = 2 \sum (6^r - 3^r)$$

$$S_n = 2 \cdot \left[\frac{6 \cdot (6^n - 1)}{5} - \frac{3 \cdot (3^n - 1)}{2} \right]$$

$$S_n = 2 \left[\frac{12(6^n - 1) - 15(3^n - 1)}{10} \right]$$

$$S_n = \frac{3}{5} [4 \cdot 6^n - 5 \cdot 3^n + 1]$$

$$\therefore n^2 - 12n + 39 = 3$$

$$n^2 - 12n + 36 = 0$$

$$n = 6$$

33. Ans. (103)

Sol. $S = 1 + 2 + 4 + 7 + \dots + T_n$

$$S = 1 + 2 + 4 + \dots$$

$$T_n = 1 + 1 + 2 + 3 + \dots + (T_n - T_{n-1})$$

$$T_n = 1 + \left(\frac{n-1}{2} \right) [2 + (n-2) \cdot 1]$$

$$T_n = 1 + 1 + \frac{n(n-1)}{2}$$

$$n = 100 \quad T_n = 1 + \frac{100 \cdot 99}{2} = 4950 + 1$$

$$n = 101 \quad T_n = 1 + \frac{101 \cdot 100}{2} = 5050 + 1 = 5051$$

$$n = 102 \quad T_n = 1 + \frac{102 \cdot 101}{2} = 5151 + 1 = 5152$$

$$n = 103 \quad T_n = 1 + \frac{103 \cdot 102}{2} = 5254$$

$$n = 104 \quad T_n = 1 + \frac{104 \cdot 103}{2} = 5357$$