STATISTICS

1. Let $a_1, a_2, \ldots.. a_{10}$ be 10 observations such that $\sum_{k=1}^{10} a_k = 50$ and $\sum_{\forall k<j} a_k \cdot a_j = 1100$. Then the standard deviation of $a_1, a_2, .., a_{10}$ is equal to :

(1) 5 (2) $\sqrt{5}$

(3) 10 (4) $\sqrt{115}$

2. The mean and standard deviation of 15 observations were found to be 12 and 3 respectively. On rechecking it was found that an observation was read as 10 in place of 12. If μ and σ^2 denote the mean and variance of the correct observations respectively, then $15(\propto + \propto^2 + \sigma^2)$ is equal to ……………….

3. If the mean and variance of the data 65, 68, 58, 44, 48, 45, 60, α, β, 60 where $\alpha > \beta$ are 56 and 66.2 respectively, then $\alpha^2 + \beta^2$ is equal to

4. If the mean and variance of five observations are $\frac{24}{5}$ and $\frac{194}{25}$ respectively and the mean of first four observations is $\frac{7}{2}$, then the variance of the first four observations is equal to

(1) $\frac{4}{5}$ (2) $\frac{77}{12}$ (3) $\frac{5}{4}$ (4) $\frac{105}{4}$

5. Let M denote the median of the following frequency distribution.

Class	0-4	4-8	8-12	12-16	16-20
Frequency	3	9	10	8	6

Then 20 M is equal to :

(1) 416 (2) 104

(3) 52 (4) 208

6. The variance σ^2 of the data

x_i	0	1	5	6	10	12	17
f_i	3	2	3	2	6	3	3

is ________.

7. Three rotten apples are accidently mixed with fifteen good apples. Assuming the random variable x to be the number of rotten apples in a draw of two apples, the variance of x is

(1) $\frac{37}{153}$ (2) $\frac{57}{153}$

(3) $\frac{47}{153}$ (4) $\frac{40}{153}$

8. Let the mean and the variance of 6 observation a, b, 68, 44, 48, 60 be 55 and 194, respectively if a > b, then a + 3b is

(1) 200 (2) 190

(3) 180 (4) 210

9. Let the median and the mean deviation about the median of 7 observation 170, 125, 230, 190, 210, a, b be 170 and $\frac{205}{7}$ respectively. Then the mean deviation about the mean of these 7 observations is:

(1) 31 (2) 28

(3) 30 (4) 32

10. Consider 10 observation $x_1, x_{2,\ldots,} x_{10}$. such that $\sum_{i=1}^{10}(x_i - \alpha) = 2$ and $\sum_{i=1}^{10}(x_i - \beta)^2 = 40$, where α, β are positive integers. Let the mean and the variance of the observations be $\frac{6}{5}$ and $\frac{84}{25}$ respectively. The $\frac{\beta}{\alpha}$ is equal to :

(1) 2 (2) $\frac{3}{2}$

(3) $\frac{5}{2}$ (4) 1

11. Let a, b, c $\in$ N and a < b < c. Let the mean, the mean deviation about the mean and the variance of the 5 observations 9, 25, a, b, c be 18, 4 and $\frac{136}{5}$, respectively. Then 2a + b – c is equal to _______ .

12. If the variance of the frequency distribution is 160, then the value of $c \in N$ is

x	c	2c	3c	4c	5c	6c
f	2	1	1	1	1	1

(1) 5 (2) 8
(3) 7 (4) 6

13. The frequency distribution of the age of students in a class of 40 students is given below.

Age	15	16	17	18	19	20
No. of Students	5	8	5	12	x	y

If the mean deviation about the median is 1.25, then 4x + 5y is equal to :

(1) 43 (2) 44
(3) 47 (4) 46

14. Let $\alpha, \beta \in R$. Let the mean and the variance of 6 observations –3, 4, 7, -6α, β be 2 and 23, respectively. The mean deviation about the mean of these 6 observations is :

(1) $\frac{13}{3}$ (2) $\frac{16}{3}$
(3) $\frac{11}{3}$ (4) $\frac{14}{3}$

15. If the mean of the following probability distribution of a random variable X;

X	0	2	4	6	8
P(X)	a	2a	a + b	2b	3b

is $\frac{46}{9}$, then the variance of the distribution is

(1) $\frac{581}{81}$ (2) $\frac{566}{81}$
(3) $\frac{173}{27}$ (4) $\frac{151}{27}$

16. Let the mean and the standard deviation of the probability distribution

X	α	1	0	–3
P(X)	$\frac{1}{3}$	K	$\frac{1}{6}$	$\frac{1}{4}$

be $\propto$ and σ, respectively. If $\sigma - \propto = 2$, then $\sigma + \propto$ is equal to ______.

17. From a lot of 10 items, which include 3 defective items, a sample of 5 items is drawn at random. Let the random variable X denote the number of defective items in the sample. If the variance of X is σ^2, then $96\sigma^2$ is equal to__________.

18. From a lot of 12 items containing 3 defectives, a sample of 5 items is drawn at random. Let the random variable X denote the number of defective items in the sample. Let items in the sample be drawn one by one without replacement. If variance of X is $\frac{m}{n}$, where gcd(m, n) = 1, then n – m is equal to _______.

19. The mean and standard deviation of 20 observations are found to be 10 and 2, respectively. On respectively, it was found that an observation by mistake was taken 8 instead of 12. The correct standard deviation is

(1) $\sqrt{3.86}$ (2) 1.8
(3) $\sqrt{3.96}$ (4) 1.94

SOLUTIONS

1. Ans. (2)

Sol. $\sum_{k=1}^{10} a_k = 50$

$a_1 + a_2 + \ldots + a_{10} = 50$ ….(i)

$\sum_{\forall k<j} a_k a_j = 1100$ ....(ii)

If $a_1 + a_2 + \ldots + a_{10} = 50$.

$(a_1 + a_2 + \ldots + a_{10})^2 = 2500$

$\Rightarrow \sum_{i=1}^{10} a_i^2 + 2\sum_{k<j} a_k a_j = 2500$

$\Rightarrow \sum_{i=1}^{10} a_i^2 = 2500 - 2(1100)$

$\sum_{i=1}^{10} a_i^2 = 300$, Standard deviation 'σ'

$= \sqrt{\frac{\sum a_i^2}{10} - \left(\frac{\sum a_i}{10}\right)^2} = \sqrt{\frac{300}{10} - \left(\frac{50}{10}\right)^2}$

$= \sqrt{30 - 25} = \sqrt{5}$

2. Ans. (2521)

Sol. Let the incorrect mean be $\propto'$ and standard deviation be σ'

We have

$\propto' = \frac{\Sigma x_i}{15} = 12 \Rightarrow \Sigma x_i = 180$

As per given information correct $\Sigma x_i = 180 - 10 + 12$

$\Rightarrow \propto$ (correct mean) $= \frac{182}{15}$

Also

$\sigma' = \sqrt{\frac{\Sigma x_i^2}{15} - 144} = 3 \Rightarrow \Sigma x_i^2 = 2295$

Correct $\Sigma x_i^2 = 2295 - 100 + 144 = 2339$

σ^2 (correct variance) $= \frac{2339}{15} - \frac{182 \cdot 182}{15 \cdot 15}$

Required value

$= 15(\propto + \propto^2 + \sigma^2)$

$= 15\left(\frac{182}{15} + \frac{182 \cdot 182}{15 \cdot 15} + \frac{2339}{15} - \frac{182 \cdot 182}{15 \cdot 15}\right)$

$= 15\left(\frac{182}{15} + \frac{2339}{15}\right)$

$= 2521$

3. Ans. (6344)

Sol. $\bar{x} = 56$

$\sigma^2 = 66.2$

$\Rightarrow \frac{\alpha^2 + \beta^2 + 25678}{10} - (56)^2 = 66.2$

$\therefore \alpha^2 + \beta^2 = 6344$

4. Ans. (3)

Sol. $\bar{X} = \frac{24}{5}; \sigma^2 = \frac{194}{25}$

Let first four observation be x_1, x_2, x_3, x_4

Here, $\frac{x_1 + x_2 + x_3 + x_4 + x_5}{5} = \frac{24}{5}$ ......(1)

Also, $\frac{x_1 + x_2 + x_3 + x_4}{4} = \frac{7}{2}$

$\Rightarrow \boxed{x_1 + x_2 + x_3 + x_4 = 14}$

Now from eqn -1

$x_5 = 10$

Now, $\sigma^2 = \frac{194}{25}$

$\frac{x_1^2 + x_2^2 + x_3^2 + x_4^2 + x_5^2}{5} - \frac{576}{25} = \frac{194}{25}$

$\Rightarrow x_1^2 + x_2^2 + x_3^2 + x_4^2 = 54$

Now, variance of first 4 observations

$\text{Var} = \frac{\sum_{i=1}^{4} x_i^2}{4} - \left(\frac{\sum_{i=1}^{4} x_i}{4}\right)^2$

$= \frac{54}{4} - \frac{49}{4} = \frac{5}{4}$

5. Ans. (4)

Sol.

Class	Frequency	Cumulative frequency
0-4	3	**3**
4-8	9	**12**
8-12	10	**22**
12-16	8	**30**
16-20	6	**36**

$M = \ell + \left(\frac{\frac{N}{2} - C}{f}\right)h$

$M = 8 + \frac{18 - 12}{10} \cdot 4$

M = 10.4

20M = 208

6. Ans. (29)

Sol.

x_i	f_i	f_ix_i	$f_ix_i^2$
0	3	0	0
1	2	2	2
5	3	15	75
6	2	12	72
10	6	60	600
12	3	36	432
17	3	51	867
	$\Sigma f_i = 22$		$\Sigma f_ix_i^2 = 2048$

$\therefore \ \Sigma f_ix_i = 176$

So $\bar{x} = \frac{\sum f_ix_i}{\sum f_i} = \frac{176}{22} = 8$

for $\sigma^2 = \frac{1}{N}\sum f_ix_i^2 - (\bar{x})^2$

$= \frac{1}{22}\cdot 2048 - (8)^2$

$= 93.090964 - 64$

$= 29.0909$

7. Ans. (4)

Sol. 3 bad apples, 15 good apples.

Let X be no of bad apples

Then $P(X = 0) = \frac{^{15}C_2}{^{18}C_2} = \frac{105}{153}$

$P(X = 1) = \frac{^{3}C_1 \cdot {^{15}C_1}}{^{18}C_2} = \frac{45}{153}$

$P(X = 2) = \frac{^{3}C_2}{^{18}C_2} = \frac{3}{153}$

$E(X) = 0\cdot\frac{105}{153} + 1\frac{45}{153} + 2\cdot\frac{3}{153} = \frac{51}{153}$

$= \frac{1}{3}$

$Var(X) = E(X^2) - (E(X))^2$

$= 0\cdot\frac{105}{153} + 1\frac{45}{153} + 4\cdot\frac{3}{153} - \left(\frac{1}{3}\right)^2$

$= \frac{57}{153} - \frac{1}{9} = \frac{40}{153}$

8. Ans. (3)

Sol. a, b, 68, 44, 48, 60

Mean = 55 $\qquad$ a > b

Variance = 194 $\qquad$ a + 3b

$\frac{a+b+68+44+48+60}{6} = 55$

$\Rightarrow 220 + a + b = 330$

$\therefore$ a + b = 110 $\qquad$ ……(1)

Also,

$\sum\frac{(x_i - \bar{x})^2}{n} = 194$

$\Rightarrow (a-55)^2 + (b-55)^2 + (68-55)^2 + (44-55)^2 + (48-55)^2 + (60-55)^2 = 194\times6$

$\Rightarrow (a-55)^2 + (b-55)^2 + 169 + 121 + 49 + 25 = 1164$

$\Rightarrow (a-55)^2 + (b-55)^2 = 1164 - 364 = 800$

$a^2 + 3025 - 110a + b^2 + 3025 - 110b = 800$

$\Rightarrow a^2 + b^2 = 800 - 6050 + 12100$

$a^2 + b^2 = 6850$ $\qquad$ ……(2)

Solve (1) & (2);

a = 75, b = 35

$\therefore a + 3b = 75 + 3(35) = 75 + 105 = 180$

9. Ans. (3)

Sol. Median = 170 $\Rightarrow$ 125, a, b, 170, 190, 210, 230

Mean deviation about

Median = $\frac{0+45+60+20+40+170-a+170-b}{7}$

$= \frac{205}{7}$

$\Rightarrow$ a + b = 300

Mean = $\frac{170+125+230+190+210+a+b}{7} = 175$

Mean deviation

About mean =

$\frac{50+175-a+175-b+5+15+35+55}{7} = 30$

10. Ans. (1)

Sol. $x_1, x_2 \ldots\ldots x_{10}$

$$\sum_{i=1}^{10}(x_i - \alpha) = 2 \Rightarrow \sum_{i=1}^{10} x_i - 10\alpha = 2$$

$$\text{Mean } \alpha = \frac{6}{5} = \frac{\Sigma x_i}{10}$$

$\therefore \quad \Sigma x_i = 12$

$10\alpha + 2 = 12 \quad \therefore \alpha = 1$

Now $\sum_{i=1}^{10}(x_i - \beta)^2 = 40$ Let $y_i = x_i - \beta$

$$\therefore \sigma_y^2 = \frac{1}{10}\sum y_i^2 - (\bar{y})^2$$

$$\sigma_x^2 = \frac{1}{10}\sum (x_i - \beta)^2 - \left(\frac{\sum_{i=1}^{10}(x_i - \beta)}{10}\right)^2$$

$$\frac{84}{25} = 4 - \left(\frac{12 - 10\beta}{10}\right)^2$$

$$\therefore \left(\frac{6 - 5\beta}{5}\right)^2 = 4 - \frac{84}{25} = \frac{16}{25}$$

$6 - 5\beta = \pm 4 \Rightarrow \beta = \frac{2}{5}$ (not possible) or $\beta = 2$

Hence $\frac{\beta}{\alpha} = 2$

11. Ans. (33)

Sol. $a, b, c \in N$ $\quad a < b < c$

$$\bar{x} = \text{mean} = \frac{9 + 25 + a + b + c}{5} = 18$$

$a + b + c = 56$

Mean deviation $= \frac{\Sigma |x_i - \bar{x}|}{n} = 4$

$= 9 + 7 + |18 - a| + |18 - b| + |18 - c| = 20$

$= |18 - a| + |18 - b| + |18 - c| = 4$

Variance $= \frac{\Sigma |x_i - \bar{x}|^2}{n} = \frac{136}{5}$

$= 81 + 49 + |18 - a|^2 + |18 - b|^2 + |18 - c|^2 = 136$

$= (18 - a)^2 + (18 - b)^2 + (18 - c)^2 = 6$

Possible values $(18 - a)^2 = 1$, $(18 - b)^2 = 1$ $(18 - c)^2 = 4$

$a < b < c$

so $\quad 18 - a = 1 \quad 18 - b = -1 \quad 18 - c = -2$

$\quad a = 17 \quad b = 19 \quad c = 20$

$a + b + c = 56$

$2a + b - c34 = 19 - 20 = 33$

12. Ans. (3)

Sol.

x	C	2C	3C	4C	5C	6C
f	2	1	1	1	1	1

$$\bar{x} = \frac{(2+2+3+4+5+6)C}{7} = \frac{22C}{7}$$

$$\text{Var}(x) = \frac{c^2\left(2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2\right)}{7}$$

$$-\left(\frac{22c}{7}\right)^2$$

$$= \frac{92c^2}{7} - c^2 \cdot \frac{484}{49}$$

$$= \frac{(644 - 484)c^2}{49} = \frac{160c^2}{49}$$

$$160 = \frac{160 \cdot c^2}{49} \Rightarrow c = 7$$

13. Ans. (2)

Sol. $x + y = 10$ ....(1)

Median = 18 = M

$$\text{M.D.} = \frac{\Sigma f_i |x_i - M|}{\Sigma f_i}$$

$$1.25 = \frac{36 + x + 2y}{40}$$

$x + 2y = 14$ ....(1)

by (1) & (2)

$x = 6$, $y = 4$

$\Rightarrow 4x + 5y = 24 + 20 = 44$

Age(x_i)	f	$\|x_i - M\|$	$f_i\|x_i - M\|$
15	5	3	15
16	8	2	16
17	5	1	5
18	12	0	0
19	x	1	x
20	y	2	2y

14. Ans. (1)

Sol. $\frac{\sum x_i}{6} = 2$ and $\frac{\sum x_i^2}{N} - \bar{x}^2 = 23$

$\alpha + \beta = 10$

$\alpha^2 + \beta^2 = 52$

solving we get $\alpha = 4, \beta = 6$

$\frac{\sum |x_i - \bar{x}|}{6} = \frac{5+2+5+8+2+4}{6} = \frac{13}{3}$

15. Ans. (2)

Sol. $\sum P_i = 1$

$a + 2a + a + b + 2b + 3b = 1$

$4a + 6b = 1$ .... (I)

$E(x) = \text{mean} = \frac{46}{9}$

$\sum P_i X_i = \frac{46}{9} \Rightarrow 4a + 4a + 4b + 12b + 24b = \frac{46}{9}$

$8a + 40b = \frac{46}{9}$

$4a + 20b = \frac{23}{9}$ .... (II)

Subtract (I) from (II) we get

$b = \frac{1}{9}$ & $a = \frac{1}{12}$

Variance $= E(x_i^2) - E(x_i)^2$

$E(x_i^2) = 0^2 \times a + 2^2 \times 2a + 4^2(a + b) + 6^2(2b) + 8^2(3b)$

$= 24a + 280b$

Put $a = \frac{1}{12}$ $b = \frac{1}{9}$

$E(x_i^2) = 2 + \frac{280}{9} = \frac{298}{9}$

$\therefore \sigma^2 = E(x_i^2) - E(x_i)^2$

$= \frac{298}{9} - \left(\frac{46}{9}\right)^2$

$\sigma^2 = \frac{298}{9} - \frac{2116}{81}$

$= \frac{566}{81}$

16. Ans. (5)

Sol. $\frac{1}{3} + k + \frac{1}{6} + \frac{1}{4} = 1 \Rightarrow k = \frac{1}{4}$

$\propto = \frac{\alpha}{3} + \frac{1}{4} - \frac{3}{4}$

$\boxed{\propto = \frac{\alpha}{3} - \frac{1}{2}}$

$\sigma = \sqrt{\left(\alpha^2 \frac{1}{3} + \frac{1}{4} + 9\frac{1}{4}\right) - \left(\frac{\alpha}{3} - \frac{1}{2}\right)^2}$

$\sigma = \sqrt{\frac{2\alpha^2}{9} + \frac{\alpha}{3} + \frac{9}{4}}$

$\sigma = \propto + 2$

$\sigma^2 = (\propto + 2)^2 \Rightarrow \frac{2\alpha^2}{9} + \frac{\alpha}{3} + \frac{9}{4} = \frac{\alpha^2}{9} + \frac{9}{4} + \alpha$

$\frac{\alpha^2}{9} - \frac{2\alpha}{3} = 0$

$\alpha = 0$, (reject) or $\alpha = 6$

($\because$ $x = 0$ is already given)

$\Rightarrow \sigma + \propto = 2\propto + 2$

$= 5$

17. Ans. (56)

Sol. X = denotes number of defective

x	0	1	2	3
P(x)	$\frac{7}{15}$	$\frac{5}{12}$	$\frac{5}{12}$	$\frac{1}{12}$
x_1^2	0	1	4	9
$P_i x_1^2$	0	$\frac{5}{12}$	$\frac{20}{12}$	$\frac{9}{12}$
$p_i x_i$	0	$\frac{5}{12}$	$\frac{10}{12}$	$\frac{3}{12}$

$\mu = \Sigma p_i x_i = \frac{18}{12}$

$\Sigma p_i x_1^2 = \frac{34}{12}$

$\sigma^2 = \Sigma p_i x_1^2 - (\mu)^2$

$= \frac{34}{12} - \left(\frac{18}{12}\right)^2 = \frac{17}{6} - \frac{9}{4}$

$\frac{34 - 27}{12} = \frac{7}{12}$

$96\sigma^2 = 96 \cdot \frac{7}{12} = 56$

18. Ans. (71)

Sol. $a = 1 - \frac{{}^{3}C_5}{{}^{12}C_5}$

$b = 3.\frac{{}^{9}C_4}{{}^{12}C_5}$

$c = 3.\frac{{}^{9}C_3}{{}^{12}C_5}$

$d = 1.\frac{{}^{9}C_2}{{}^{12}C_5}$

u = 0.a + 1.b + 2.c + 3.d = 1.25

σ^2 = 0.a +1.b + 4.c + 9d – u^2

$\sigma^2 = \frac{105}{176}$

Ans. 176 – 105 = 71

19. Ans. (3)

Sol. Mean $(\bar{x}) = 10$

$\Rightarrow \frac{\Sigma x_i}{20} = 10$

Σx_i = 10×20 = 200

If 8 is replaced by 12, then Σx_i = 200 – 8 + 12 = 204

$\therefore$ Correct mean $(\bar{x}) = \frac{\Sigma x_i}{20}$

$= \frac{204}{20} = 10.2$

$\because$ Standard deviation = 2

$\therefore$ Variance = $(S.D.)^2 = 2^2 = 4$

$\Rightarrow \frac{\Sigma x_i^2}{20} - \left(\frac{\Sigma x_i}{20}\right)^2 = 4$

$\Rightarrow \frac{\Sigma x_i^2}{20} - (10)^2 = 4$

$\Rightarrow \frac{\Sigma x_i^2}{20} = 104$

$\Rightarrow \Sigma x_i^2 = 2080$

Now, replaced '8' observations by '12'

Then, $\Sigma x_i^2 = 2080 - 8^2 + 12^2 = 2160$

$\therefore$ Variance of removing observations

$\Rightarrow \frac{\Sigma x_i^2}{20} - \left(\frac{\Sigma x_i}{20}\right)^2$

$\Rightarrow \frac{2160}{20} - (10.2)^2$

$\Rightarrow$ 108 – 104.04

$\Rightarrow$ 3.96

Correct standard deviation

$= \sqrt{3.96}$