TRIGONOMETRIC EQUATION

1. If $2\tan^2\theta - 5\sec\theta = 1$ has exactly 7 solutions in the interval $\left[0, \frac{n\pi}{2}\right]$, for the least value of $n \in N$ then $\sum_{k=1}^{n} \frac{k}{2^k}$ is equal to :

(1) $\frac{1}{2^{15}}(2^{14} - 14)$ (2) $\frac{1}{2^{14}}(2^{15} - 15)$

(3) $1 - \frac{15}{2^{13}}$ (4) $\frac{1}{2^{13}}(2^{14} - 15)$

2. If $\alpha, -\frac{\pi}{2} < \alpha < \frac{\pi}{2}$ is the solution of $4\cos\theta + 5\sin\theta = 1$, then the value of $\tan\alpha$ is

(1) $\frac{10 - \sqrt{10}}{6}$ (2) $\frac{10 - \sqrt{10}}{12}$

(3) $\frac{\sqrt{10} - 10}{12}$ (4) $\frac{\sqrt{10} - 10}{6}$

3. If $2\sin^3x + \sin 2x \cos x + 4\sin x - 4 = 0$ has exactly 3 solutions in the interval $\left[0, \frac{n\pi}{2}\right]$, $n \in N$, then the roots of the equation $x^2 + nx + (n - 3) = 0$ belong to :

(1) $(0, \infty)$ (2) $(-\infty, 0)$

(3) $\left(-\frac{\sqrt{17}}{2}, \frac{\sqrt{17}}{2}\right)$ (4) Z

4. The number of solutions, of the equation $e^{\sin x} - 2e^{-\sin x} = 2$ is

(1) 2

(2) more than 2

(3) 1

(4) 0

5. The number of solutions of the equation $4\sin^2x - 4\cos^3x + 9 - 4\cos x = 0$; $x \in [-2\pi, 2\pi]$ is:

(1) 1

(2) 3

(3) 2

(4) 0

6. Let $\left|\cos\theta\cos(60 - \theta)\cos(60 - \theta)\right| \le \frac{1}{8}, \theta \in [0, 2\pi]$

Then, the sum of all $\theta \in [0, 2\pi]$, where $\cos 3\theta$ attains its maximum value, is :

(1) 9π

(2) 18π

(3) 6π

(4) 15π

7. The number of solutions of $\sin^2x + (2 + 2x - x^2)\sin x - 3(x - 1)^2 = 0$, where $-\pi \le x \le \pi$, is

SOLUTIONS

1. Ans. (4)

Sol. $2\tan^2\theta - 5\sec\theta - 1 = 0$

$\Rightarrow 2\sec^2\theta - 5\sec\theta - 3 = 0$

$\Rightarrow (2\sec\theta + 1)(\sec\theta - 3) = 0$

$\Rightarrow \sec\theta = -\frac{1}{2}, 3$

$\Rightarrow \cos\theta = -2, \frac{1}{3}$

$\Rightarrow \cos\theta = \frac{1}{3}$

For 7 solutions n = 13

So, $\sum_{k=1}^{13} \frac{k}{2^k} = S$ (say)

$S = \frac{1}{2} + \frac{2}{2^2} + \frac{3}{2^3} + ... + \frac{13}{2^{13}}$

$\frac{1}{2}S = \frac{1}{2^2} + \frac{1}{2^3} + ... + \frac{12}{2^{13}} + \frac{13}{2^{14}}$

$\Rightarrow \frac{S}{2} = \frac{1}{2} \cdot \frac{1 - \frac{1}{2^{13}}}{1 - \frac{1}{2}} - \frac{13}{2^{14}} \Rightarrow S = 2.\left(\frac{2^{13} - 1}{2^{13}}\right) - \frac{13}{2^{13}}$

2. Ans. (3)

Sol. $4 + 5\tan\theta = \sec\theta$

Squaring : $24\tan^2\theta + 40\tan\theta + 15 = 0$

$\tan\theta = \frac{-10 \pm \sqrt{10}}{12}$

and $\tan\theta = -\left(\frac{10 + \sqrt{10}}{12}\right)$ is Rejected.

(3) is correct.

3. Ans. (2)

Sol. $2\sin^3 x + 2\sin x.\cos^2 x + 4\sin x - 4 = 0$

$2\sin^3 x + 2\sin x(1 - \sin^2 x) + 4\sin x - 4 = 0$

$6\sin x - 4 = 0$

$\sin x = \frac{2}{3}$

n = 5 (in the given interval)

$x^2 + 5x + 2 = 0$

$x = \frac{-5 \pm \sqrt{17}}{2}$

Required interval $(-\infty, 0)$

4. Ans. (4)

Sol. Take $e^{\sin x} = t\ (t > 0)$

$\Rightarrow t - \frac{2}{t} = 2$

$\Rightarrow \frac{t^2 - 2}{t} = 2$

$\Rightarrow t^2 - 2t - 2 = 0$

$\Rightarrow t^2 - 2t + 1 = 3$

$\Rightarrow (t - 1)^2 = 3$

$\Rightarrow t = 1 \pm \sqrt{3}$

$\Rightarrow t = 1 \pm 1.73$

$\Rightarrow t = 2.73$ or -0.73 (rejected as t > 0)

$\Rightarrow e^{\sin x} = 2.73$

$\Rightarrow \log_e e^{\sin x} = \log_e 2.73$

$\Rightarrow \sin x = \log_e 2.73 > 1$

So no solution.

5. Ans. (4)

Sol. $4\sin^2x - 4\cos^3x + 9 - 4\cos x = 0$; $x \in [-2\pi, 2\pi]$

$4 - 4\cos^2x - 4\cos^3x + 9 - 4\cos x = 0$

$4\cos^3x + 4\cos^2x + 4\cos x - 13 = 0$

$4\cos^3x + 4\cos^2x + 4\cos x = 13$

L.H.S. ≤ 12 can't be equal to 13.

6. Ans. (3)

Sol. We know that

$(\cos\theta)(\cos(60° - \theta))(\cos(60° + \theta)) = \frac{1}{4}\cos 3\theta$

So equation reduces to $\left|\frac{1}{4}\cos 3\theta\right| \leq \frac{1}{8}$

$\Rightarrow |\cos 3\theta| \leq \frac{1}{2}$

$\Rightarrow -\frac{1}{2} \leq \cos 3\theta \leq \frac{1}{2}$

$\Rightarrow$ maximum value of $\cos 3\theta = \frac{1}{2}$, here

$\Rightarrow 3\theta = 2n\pi \pm \frac{\pi}{3}$

$\theta = \frac{2n\pi}{3} \pm \frac{\pi}{9}$

As $\theta \in [0, 2\pi]$ possible values are

$\theta = \left\{\frac{\pi}{9}, \frac{5\pi}{9}, \frac{7\pi}{9}, \frac{11\pi}{9}, \frac{13\pi}{9}, \frac{17\pi}{9}\right\}$

Whose sum is

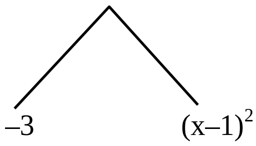
$\frac{\pi}{9} + \frac{5\pi}{9} + \frac{7\pi}{9} + \frac{11\pi}{9} + \frac{13\pi}{9} + \frac{17\pi}{9} = \frac{54\pi}{9} = 6\pi$

7. Ans. (2)

Sol. $\sin^2x - (x^2 - 2x - 2)\sin x - 3(x-1)^2 = 0$

$\sin^2x - (x-1)^2)\sin x - 3(x-1)^2 = 0$

roots :



$\sin x = -3$ (reject) or $(x-1)^2$

$\sin x = (x-1)^2$