

3D COORDINATE GEOMETRY

1. The distance, of the point $(7, -2, 11)$ from the line $\frac{x-6}{1} = \frac{y-4}{0} = \frac{z-8}{3}$ along the line $\frac{x-5}{2} = \frac{y-1}{-3} = \frac{z-5}{6}$, is :
- (1) 12 (2) 14
 (3) 18 (4) 21
2. If the shortest distance between the lines $\frac{x-4}{1} = \frac{y+1}{2} = \frac{z}{-3}$ and $\frac{x-\lambda}{2} = \frac{y+1}{4} = \frac{z-2}{-5}$ is $\frac{6}{\sqrt{5}}$, then the sum of all possible values of λ is :
- (1) 5 (2) 8
 (3) 7 (4) 10
3. Let the image of the point $(1, 0, 7)$ in the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$ be the point (α, β, γ) . Then which one of the following points lies on the line passing through (α, β, γ) and making angles $\frac{2\pi}{3}$ and $\frac{3\pi}{4}$ with y-axis and z-axis respectively and an acute angle with x-axis ?
- (1) $(1, -2, 1 + \sqrt{2})$ (2) $(1, 2, 1 - \sqrt{2})$
 (3) $(3, 4, 3 - 2\sqrt{2})$ (4) $(3, -4, 3 + 2\sqrt{2})$
4. The lines $\frac{x-2}{2} = \frac{y}{-2} = \frac{z-7}{16}$ and $\frac{x+3}{4} = \frac{y+2}{3} = \frac{z+2}{1}$ intersect at the point P. If the distance of P from the line $\frac{x+1}{2} = \frac{y-1}{3} = \frac{z-1}{1}$ is l , then $14l^2$ is equal to.....
5. Let PQR be a triangle with $R(-1, 4, 2)$. Suppose $M(2, 1, 2)$ is the mid point of PQ. The distance of the centroid of ΔPQR from the point of intersection of the line $\frac{x-2}{0} = \frac{y}{2} = \frac{z+3}{-1}$ and $\frac{x-1}{1} = \frac{y+3}{-3} = \frac{z+1}{1}$ is
- (1) 69 (2) 9 (3) $\sqrt{69}$ (4) $\sqrt{99}$
6. A line with direction ratios 2, 1, 2 meets the lines $x = y + 2 = z$ and $x + 2 = 2y = 2z$ respectively at the point P and Q. If the length of the perpendicular from the point $(1, 2, 12)$ to the line PQ is l , then l^2 is
7. Let $P(3, 2, 3)$, $Q(4, 6, 2)$ and $R(7, 3, 2)$ be the vertices of ΔPQR . Then, the angle $\angle QPR$ is
- (1) $\frac{\pi}{6}$ (2) $\cos^{-1}\left(\frac{7}{18}\right)$
 (3) $\cos^{-1}\left(\frac{1}{18}\right)$ (4) $\frac{\pi}{3}$
8. Let O be the origin, and M and N be the points on the lines $\frac{x-5}{4} = \frac{y-4}{1} = \frac{z-5}{3}$ and $\frac{x+8}{12} = \frac{y+2}{5} = \frac{z+11}{9}$ respectively such that MN is the shortest distance between the given lines. Then $OM \cdot ON$ is equal to _____.
9. Let (α, β, γ) be the foot of perpendicular from the point $(1, 2, 3)$ on the line $\frac{x+3}{5} = \frac{y-1}{2} = \frac{z+4}{3}$. then $19(\alpha + \beta + \gamma)$ is equal to :
- (1) 102 (2) 101
 (3) 99 (4) 100
10. If d_1 is the shortest distance between the lines $x + 1 = 2y = -12z$, $x = y + 2 = 6z - 6$ and d_2 is the shortest distance between the lines $\frac{x-1}{2} = \frac{y+8}{-7} = \frac{z-4}{5}$, $\frac{x-1}{2} = \frac{y-2}{1} = \frac{z-6}{-3}$, then the value of $\frac{32\sqrt{3}d_1}{d_2}$ is :
11. Let a line passing through the point $(-1, 2, 3)$ intersect the lines $L_1: \frac{x-1}{3} = \frac{y-2}{2} = \frac{z+1}{-2}$ at $M(\alpha, \beta, \gamma)$ and $L_2: \frac{x+2}{-3} = \frac{y-2}{-2} = \frac{z-1}{4}$ at $N(a, b, c)$. Then the value of $\frac{(\alpha + \beta + \gamma)^2}{(a + b + c)^2}$ equals _____.

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12. The distance of the point Q(0, 2, -2) from the line passing through the point P(5, -4, 3) and perpendicular to the lines
- $$\vec{r} = (-3\hat{i} + 2\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 5\hat{k}), \lambda \in \mathbb{R} \text{ and}$$
- $$\vec{r} = (\hat{i} - 2\hat{j} + \hat{k}) + \mu(-\hat{i} + 3\hat{j} + 2\hat{k}), \mu \in \mathbb{R} \text{ is}$$
- (1) $\sqrt{86}$ (2) $\sqrt{20}$
 (3) $\sqrt{54}$ (4) $\sqrt{74}$
13. Let Q and R be the feet of perpendiculars from the point P(a, a, a) on the lines $x = y, z = 1$ and $x = -y, z = -1$ respectively. If $\angle QPR$ is a right angle, then $12a^2$ is equal to _____
14. Let (α, β, γ) be mirror image of the point (2, 3, 5) in the line $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$. Then $2\alpha + 3\beta + 4\gamma$ is equal to
- (1) 32 (2) 33
 (3) 31 (4) 34
15. The shortest distance between lines L_1 and L_2 , where $L_1: \frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+4}{2}$ and L_2 is the line passing through the points A(-4, 4, 3), B(-1, 6, 3) and perpendicular to the line $\frac{x-3}{-2} = \frac{y}{3} = \frac{z-1}{1}$, is
- (1) $\frac{121}{\sqrt{221}}$ (2) $\frac{24}{\sqrt{117}}$
 (3) $\frac{141}{\sqrt{221}}$ (4) $\frac{42}{\sqrt{117}}$
16. A line passes through A(4, -6, -2) and B(16, -2, 4). The point P(a, b, c) where a, b, c are non-negative integers, on the line AB lies at a distance of 21 units, from the point A. The distance between the points P(a, b, c) and Q(4, -12, 3) is equal to _____.
17. If the shortest distance between the lines $\frac{x-\lambda}{-2} = \frac{y-2}{1} = \frac{z-1}{1}$ and $\frac{x-\sqrt{3}}{1} = \frac{y-1}{-2} = \frac{z-2}{1}$ is 1, then the sum of all possible values of λ is :
- (1) 0 (2) $2\sqrt{3}$
 (3) $3\sqrt{3}$ (4) $-2\sqrt{3}$
18. Let P and Q be the points on the line $\frac{x+3}{8} = \frac{y-4}{2} = \frac{z+1}{2}$ which are at a distance of 6 units from the point R (1, 2, 3). If the centroid of the triangle PQR is (α, β, γ) , then $\alpha^2 + \beta^2 + \gamma^2$ is:
- (1) 26 (2) 36
 (3) 18 (4) 24
19. If the mirror image of the point P(3, 4, 9) in the line $\frac{x-1}{3} = \frac{y+1}{2} = \frac{z-2}{1}$ is (α, β, γ) , then $14(\alpha + \beta + \gamma)$ is :
- (1) 102 (2) 138
 (3) 108 (4) 132
20. If the shortest distance between the lines $\frac{x-\lambda}{2} = \frac{y-4}{3} = \frac{z-3}{4}$ and $\frac{x-2}{4} = \frac{y-4}{6} = \frac{z-7}{8}$ is $\frac{13}{\sqrt{29}}$, then a value of λ is :
- (1) $-\frac{13}{25}$ (2) $\frac{13}{25}$
 (3) 1 (4) -1
21. Let $P(\alpha, \beta, \gamma)$ be the image of the point Q(1, 6, 4) in the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$. Then $2\alpha + \beta + \gamma$ is equal to _____.
22. Consider the line L passing through the points (1, 2, 3) and (2, 3, 5). The distance of the point $\left(\frac{11}{3}, \frac{11}{3}, \frac{19}{3}\right)$ from the line L along the line $\frac{3x-11}{2} = \frac{3y-11}{1} = \frac{3z-19}{2}$ is equal to :
- (1) 3 (2) 5
 (3) 4 (4) 6

23. The square of the distance of the image of the point $(6, 1, 5)$ in the line $\frac{x-1}{3} = \frac{y}{2} = \frac{z-2}{4}$, from the origin is _____.
24. Let the line L intersect the lines $x-2 = -y = z-1$, $2(x+1) = 2(y-1) = z+1$ and be parallel to the line $\frac{x-2}{3} = \frac{y-1}{1} = \frac{z-2}{2}$. Then which of the following points lies on L ?
- (1) $\left(-\frac{1}{3}, 1, 1\right)$ (2) $\left(-\frac{1}{3}, 1, -1\right)$
- (3) $\left(-\frac{1}{3}, -1, -1\right)$ (4) $\left(-\frac{1}{3}, -1, 1\right)$
25. The shortest distance between the line $\frac{x-3}{4} = \frac{y+7}{-11} = \frac{z-1}{5}$ and $\frac{x-5}{3} = \frac{y-9}{-6} = \frac{z+2}{1}$ is :
- (1) $\frac{187}{\sqrt{563}}$ (2) $\frac{178}{\sqrt{563}}$
- (3) $\frac{185}{\sqrt{563}}$ (4) $\frac{179}{\sqrt{563}}$
26. Let the point, on the line passing through the points $P(1, -2, 3)$ and $Q(5, -4, 7)$, farther from the origin and at a distance of 9 units from the point P, be (α, β, γ) . Then $\alpha^2 + \beta^2 + \gamma^2$ is equal to :
- (1) 155 (2) 150
- (3) 160 (4) 165
27. If the shortest distance between the lines $\frac{x+2}{2} = \frac{y+3}{3} = \frac{z-5}{4}$ and $\frac{x-3}{1} = \frac{y-2}{-3} = \frac{z+4}{2}$ is $\frac{38}{3\sqrt{5}}k$ and $\int_0^k [x^2] dx = \alpha - \sqrt{\alpha}$, where $[x]$ denotes the greatest integer function, then $6\alpha^3$ is equal to _____.
28. Let (α, β, γ) be the image of the point $(8, 5, 7)$ in the line $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-2}{5}$. Then $\alpha + \beta + \gamma$ is equal to
- (1) 16 (2) 18
- (3) 14 (4) 20
29. Let the point $(-1, \alpha, \beta)$ lie on the line of the shortest distance between the lines $\frac{x+2}{-3} = \frac{y-2}{4} = \frac{z-5}{2}$ and $\frac{x+2}{-1} = \frac{y+6}{2} = \frac{z-1}{0}$. Then $(\alpha - \beta)^2$ is equal to _____.
30. Let P the point of intersection of the lines $\frac{x-2}{1} = \frac{y-4}{5} = \frac{z-2}{1}$ and $\frac{x-3}{2} = \frac{y-2}{3} = \frac{z-3}{2}$. Then, the shortest distance of P from the line $4x = 2y = z$ is
- (1) $\frac{5\sqrt{4}}{7}$ (2) $\frac{\sqrt{14}}{7}$
- (3) $\frac{3\sqrt{4}}{7}$ (4) $\frac{6\sqrt{4}}{7}$
31. Consider a line L passing through the points $P(1, 2, 1)$ and $Q(2, 1, -1)$. If the mirror image of the point $A(2, 2, 2)$ in the line L is (α, β, γ) , then $\alpha + \beta + 6\gamma$ is equal to
32. Let d be the distance of the point of intersection of the lines $\frac{x+6}{3} = \frac{y}{2} = \frac{z+1}{1}$ and $\frac{x-7}{4} = \frac{y-9}{3} = \frac{z-4}{2}$ from the point $(7, 8, 9)$. Then $d^2 + 6$ is equal to :
- (1) 72 (2) 69
- (3) 75 (4) 78
33. Let P (α, β, γ) be the image of the point $Q(3, -3, 1)$ in the line $\frac{x-0}{1} = \frac{y-3}{1} = \frac{z-1}{-1}$ and R be the point $(2, 5, -1)$. If the area of the triangle PQR is λ and $\lambda^2 = 14K$, then K is equal to:
- (1) 36 (2) 72
- (3) 18 (4) 81

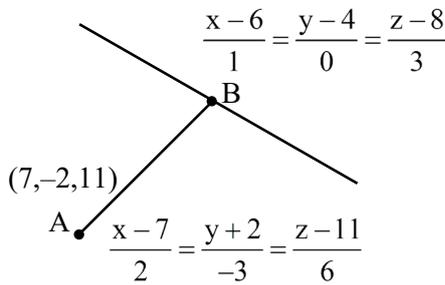
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34. If the shortest distance between the lines $\frac{x-\lambda}{3} = \frac{y-2}{-1} = \frac{z-1}{1}$ and $\frac{x+2}{-3} = \frac{y+5}{2} = \frac{z-4}{4}$ is $\frac{44}{\sqrt{30}}$, then the largest possible value of $|\lambda|$ is equal to _____.
35. If the line $\frac{2-x}{3} = \frac{3y-2}{4\lambda+1} = 4-z$ makes a right angle with the line $\frac{x+3}{3\mu} = \frac{1-2y}{6} = \frac{5-z}{7}$, then $4\lambda + 9\mu$ is equal to :
 (1) 13 (2) 4
 (3) 5 (4) 6
36. Let P(x, y, z) be a point in the first octant, whose projection in the xy-plane is the point Q. Let $OP = \gamma$; the angle between OQ and the positive x-axis be θ ; and the angle between OP and the positive z-axis be φ , where O is the origin. Then the distance of P from the x-axis is :
 (1) $\gamma\sqrt{1 - \sin^2 \varphi \cos^2 \theta}$ (2) $\gamma\sqrt{1 + \cos^2 \theta \sin^2 \varphi}$
 (3) $\gamma\sqrt{1 - \sin^2 \theta \cos^2 \varphi}$ (4) $\gamma\sqrt{1 + \cos^2 \varphi \sin^2 \theta}$
37. If the shortest distance between the lines $L_1: \vec{r} = (2 + \lambda)\hat{i} + (1 - 3\lambda)\hat{j} + (3 + 4\lambda)\hat{k}, \lambda \in \mathbb{R}$ and $L_2: \vec{r} = 2(1 + \alpha)\hat{i} + 3(1 + \alpha)\hat{j} + (5 + \alpha)\hat{k}, \alpha \in \mathbb{R}$ is $\frac{m}{\sqrt{n}}$, where $\gcd(m, n) = 1$, then the value of $m + n$ equals.
 (1) 384
 (2) 387
 (3) 377
 (4) 390
38. The shortest distance between the lines $\frac{x-3}{2} = \frac{y+15}{-7} = \frac{z-9}{5}$ and $\frac{x+1}{2} = \frac{y-1}{1} = \frac{z-9}{-3}$ is
 (1) $6\sqrt{3}$ (2) $4\sqrt{3}$
 (3) $5\sqrt{3}$ (4) $8\sqrt{3}$
39. Let P be the point (10, -2, -1) and Q be the foot of the perpendicular drawn from the point R(1, 7, 6) on the line passing through the points (2, -5, 11) and (-6, 7, -5). Then the length of the line segment PQ is equal to _____.

SOLUTIONS

1. **Ans. (2)**

Sol. $B = (2\lambda + 7, -3\lambda - 2, 6\lambda + 11)$



Point B lies on $\frac{x-6}{1} = \frac{y-4}{0} = \frac{z-8}{3}$

$$\frac{2\lambda + 7 - 6}{1} = \frac{-3\lambda - 2 - 4}{0} = \frac{6\lambda + 11 - 8}{3}$$

$$-3\lambda - 6 = 0$$

$$\lambda = -2$$

$$B \Rightarrow (3, 4, -1)$$

$$AB = \sqrt{(7-3)^2 + (-2-4)^2 + (11-1)^2}$$

$$= \sqrt{16 + 36 + 144}$$

$$= \sqrt{196} = 14$$

2. **Ans. (2)**

Sol. $\frac{x-4}{1} = \frac{y+1}{2} = \frac{z}{-3}$

$$\frac{x-\lambda}{2} = \frac{y+1}{4} = \frac{z-2}{-5}$$

the shortest distance between the lines

$$= \frac{|(a-b) \cdot (d_1 \times d_2)|}{|d_1 \cdot d_2|}$$

$$= \frac{\begin{vmatrix} \lambda-4 & 0 & 2 \\ 1 & 2 & -3 \\ 2 & 4 & -5 \end{vmatrix}}{\begin{vmatrix} 1 & 2 & -3 \\ 2 & 4 & -5 \end{vmatrix}}$$

$$= \frac{(\lambda-4)(-10+12) - 0 + 2(4-4)}{|2\hat{i} - 1\hat{j} + 0\hat{k}|}$$

$$\frac{6}{\sqrt{5}} = \left| \frac{2(\lambda-4)}{\sqrt{5}} \right|$$

$$3 = |\lambda - 4|$$

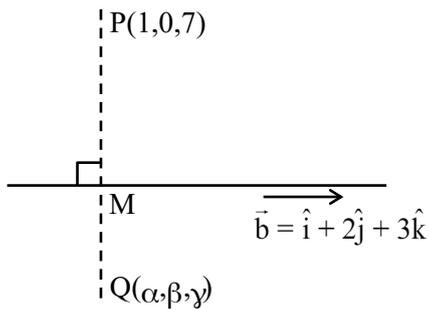
$$\lambda - 4 = \pm 3$$

$$\lambda = 7, 1$$

Sum of all possible values of λ is = 8

3. **Ans. (3)**

Sol. $L_1 = \frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3} = \lambda$



$$M(\lambda-1, 2+2\lambda, 3\lambda-5)$$

$$PM = (\lambda-1)\hat{i} + (1+2\lambda)\hat{j} + (3\lambda-5)\hat{k}$$

PM is perpendicular to line L_1

$$PM \cdot b = 0 \quad (b = \hat{i} + 2\hat{j} + 3\hat{k})$$

$$\Rightarrow \lambda - 1 + 4\lambda + 2 + 9\lambda - 15 = 0$$

$$14\lambda = 14 \Rightarrow \lambda = 1$$

$$\therefore M(1, 3, 5)$$

$$Q = 2MP \quad [M \text{ is midpoint of } P \text{ \& } Q]$$

$$Q = 2\hat{i} + 6\hat{j} + 10\hat{k} - \hat{i} - 7\hat{k}$$

$$Q = \hat{i} + 6\hat{j} + 3\hat{k}$$

$$\therefore (\alpha, \beta, \gamma) = (1, 6, 3)$$

Required line having direction cosine (l, m, n)

$$l^2 + m^2 + n^2 = 1$$

$$\Rightarrow l^2 - \left(\frac{1}{2}\right)^2 + \left(-\frac{1}{\sqrt{2}}\right)^2 = 1$$

$$l^2 = \frac{1}{4}$$

$$\therefore l = \frac{1}{2} \text{ [Line make acute angle with x-axis]}$$

Equation of line passing through $(1, 6, 3)$ will be

$$r = (\hat{i} + 6\hat{j} + 3\hat{k}) + \alpha \left(\frac{1}{2}\hat{i} - \frac{1}{2}\hat{j} - \frac{1}{\sqrt{2}}\hat{k} \right)$$

Option (3) satisfying for $\alpha = 4$

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4. Ans. (108)

Sol. $\frac{x-2}{1} = \frac{y}{-1} = \frac{z-7}{8} = \lambda$

$$\frac{x+3}{4} = \frac{y+2}{3} = \frac{z+2}{1} = k$$

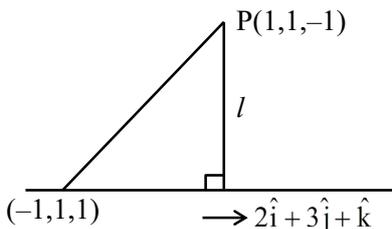
$$\Rightarrow \lambda + 2 = 4k - 3$$

$$-\lambda = 3k - 2$$

$$\Rightarrow k = 1, \lambda = -1$$

$$8\lambda + 7 = k - 2$$

$$\therefore \underline{P} (1, 1, -1)$$


 Projection of $2\hat{i} - 2\hat{k}$ on $2\hat{i} + 3\hat{j} + \hat{k}$ is

$$= \frac{4 - 2}{\sqrt{4 + 9 + 1}} = \frac{2}{\sqrt{14}}$$

$$\therefore l^2 = 8 - \frac{4}{14} = \frac{108}{14}$$

$$\Rightarrow 14l^2 = 108$$

5. Ans. (3)

Sol. Centroid G divides MR in 1 : 2

$$G(1, 2, 2)$$

Point of intersection A of given lines is (2, -6, 0)

$$AG = \sqrt{69}$$

6. Ans. (65)

Sol. Let P(t, t-2, t) and Q(2s-2, s, s)

D.R's of PQ are 2, 1, 2

$$\frac{2s-2-t}{2} = \frac{s-t+2}{1} = \frac{s-t}{2}$$

$$\Rightarrow t = 6 \text{ and } s = 2$$

$$\Rightarrow P(6, 4, 6) \text{ and } Q(2, 2, 2)$$

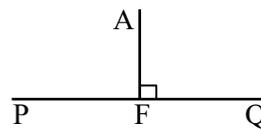
$$PQ: \frac{x-2}{2} = \frac{y-2}{1} = \frac{z-2}{2} = \lambda$$

Let F(2λ+2, λ+2, 2λ+2)

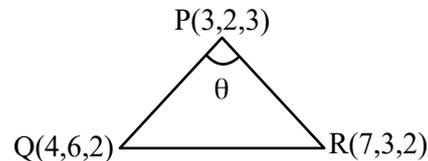
$$\underline{A}(1, 2, 12)$$

$$AF \cdot PQ = 0$$

$$\therefore \lambda = 2$$

 So F(6, 4, 6) and $AF = \sqrt{65}$

7. Ans. (4)

Sol.



Direction ratio of PR = (4, 1, -1)

Direction ratio of PQ = (1, 4, -1)

$$\text{Now, } \cos \theta = \frac{|4 + 4 + 1|}{\sqrt{18} \cdot \sqrt{18}}$$

$$\theta = \frac{\pi}{3}$$

8. Ans. (9)

Sol. $L_1: \frac{x-5}{4} = \frac{y-4}{1} = \frac{z-5}{3} = \lambda$ drs (4, 1, 3) = b_1

$$M(4\lambda + 5, \lambda + 4, 3\lambda + 5)$$

$$L_2: \frac{x+8}{12} = \frac{y+2}{5} = \frac{z+11}{9} = \infty$$

$$\underline{N}(12\infty - 8, 5\infty - 2, 9\infty - 11)$$

$$MN = (4\lambda - 12\infty + 13, \lambda - 5\infty + 6, 3\lambda - 9\infty + 16) \dots (1)$$

Now

$$\hat{a} \cdot \hat{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 1 & 3 \\ 12 & 5 & 9 \end{vmatrix} = -6\hat{i} + 8\hat{k} \dots (2)$$

Equation (1) and (2)

$$\therefore \frac{4\lambda - 12\infty + 13}{-6} = \frac{\lambda - 5\infty + 6}{0} = \frac{3\lambda - 9\infty + 16}{8}$$

I and II

$$\lambda - 5\infty + 6 = 0 \dots (3)$$

I and III

$$\lambda - 3\infty + 4 = 0 \dots (4)$$

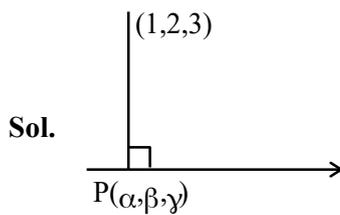
Solve (3) and (4) we get

$$\lambda = -1, \infty = 1$$

$$\therefore M(1, 3, 2)$$

$$\underline{N}(4, 3, -2)$$

$$\therefore OM \cdot ON = 4 + 9 - 4 = 9$$

9. Ans. (2)

 Let foot P $(5k - 3, 2k + 1, 3k - 4)$

 DR 's \rightarrow AP : $5k - 4, 2k - 1, 3k - 7$

 DR 's \rightarrow Line: $5, 2, 3$

Condition of perpendicular lines

$$(25k - 20) + (4k - 2) + (9k - 21) = 0$$

$$\text{Then } k = \frac{43}{38}$$

$$\text{Then } 19(\alpha + \beta + \gamma) = 101$$

10. Ans. (16)

Sol. $L_1: \frac{x+1}{1} = \frac{y}{1/2} = \frac{z}{-1/12},$

$$L_2: \frac{x}{1} = \frac{y+2}{1} = \frac{z-1}{6}$$

 $d_1 =$ shortest distance between L_1 & L_2

$$= \frac{\begin{vmatrix} \bullet & \bullet & \bullet & \bullet \\ (a_2 - a_1) & (b_1 - b_2) & (c_1 - c_2) & d_2 - d_1 \\ (b_1 - b_2) & (c_1 - c_2) & d_2 - d_1 & 0 \end{vmatrix}}{\begin{vmatrix} (b_1 - b_2) & (c_1 - c_2) & d_2 - d_1 \\ (c_1 - c_2) & d_2 - d_1 & 0 \\ d_2 - d_1 & 0 & 0 \end{vmatrix}}$$

$$d_1 = 2$$

$$L_3: \frac{x-1}{2} = \frac{y+8}{-7} = \frac{z-4}{5}, \quad L_4: \frac{x-1}{2} = \frac{y-2}{1} = \frac{z-6}{-3}$$

 $d_2 =$ shortest distance between L_3 & L_4

$$d_2 = \frac{12}{\sqrt{3}} \text{ Hence}$$

$$= \frac{32\sqrt{3}d_1}{d_2} = \frac{32\sqrt{3} \cdot 2}{\frac{12}{\sqrt{3}}} = 16$$

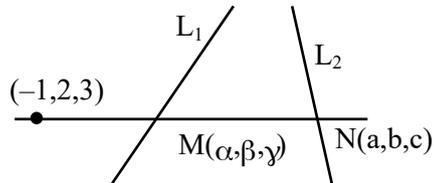
11. Ans. (196)

Sol. $M(\beta\lambda + 1, 2\lambda + 2, -2\lambda - 1)$

$$\therefore \alpha + \beta + \gamma = 3\lambda + 2$$

$$N(-3\alpha - 2, -2\alpha + 2, 4\alpha + 1)$$

$$\therefore a + b + c = -\alpha + 1$$



$$\frac{3\lambda + 2}{-3\alpha - 1} = \frac{2\lambda}{-2\alpha} = \frac{-2\lambda - 4}{4\alpha - 2}$$

$$3\lambda\alpha + 2\alpha = 2\lambda\alpha + \lambda$$

$$2\alpha = \lambda$$

$$2\lambda\alpha - \lambda = \lambda\alpha + 2\alpha$$

$$\lambda\alpha = \lambda + 2\alpha$$

$$\Rightarrow \lambda\alpha = 2\lambda$$

$$\Rightarrow \alpha = 2 \quad (\lambda \neq 0)$$

$$\therefore \lambda = 4$$

$$\alpha + \beta + \gamma = 14$$

$$a + b + c = -1$$

$$\frac{(\alpha + \beta + \gamma)^2}{(a + b + c)^2} = 196$$

12. Ans. (4)

Sol. A vector in the direction of the required line can be obtained by cross product of

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 5 \\ -1 & 3 & 2 \end{vmatrix}$$

$$= -9\hat{i} - 9\hat{j} + 9\hat{k}$$

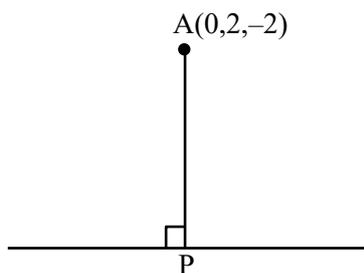
Required line,

$$\bullet \quad r = (5\hat{i} - 4\hat{j} + 3\hat{k}) + \lambda'(-9\hat{i} - 9\hat{j} + 9\hat{k})$$

$$\bullet \quad r = (5\hat{i} - 4\hat{j} + 3\hat{k}) + \lambda(\hat{i} + \hat{j} - \hat{k})$$

3D COORDINATE GEOMETRY

Now distance of (0, 2, -2)



$$\text{P.V. of P} \equiv (5 + \lambda)\hat{i} + (\lambda - 4)\hat{j} + (3 - \lambda)\hat{k}$$

$$\bullet \text{ AP} = (5 + \lambda)\hat{i} + (\lambda - 6)\hat{j} + (5 - \lambda)\hat{k}$$

$$\bullet \text{ AP} \cdot (\hat{i} + \hat{j} - \hat{k}) = 0$$

$$5 + \lambda + \lambda - 6 - 5 + \lambda = 0$$

$$\lambda = 2$$

$$\bullet |AP| = \sqrt{49 + 16 + 9}$$

$$\bullet |AP| = \sqrt{74}$$

13. Ans. (12)

$$\text{Sol. } \frac{x}{1} = \frac{y}{1} = \frac{z-1}{0} = r \rightarrow Q(r, r, 1)$$

$$\frac{x}{1} = \frac{y}{-1} = \frac{z+1}{0} = k \rightarrow R(k, -k, -1)$$

$$\overline{PQ} = (a-r)\hat{i} + (a-r)\hat{j} + (a-1)\hat{k}$$

$$a = r + a - r = 0.$$

$$2a = 2r \rightarrow a = r$$

$$\bullet \bullet \bullet \bullet \text{ PR} = (a-k)\hat{j} + (a+k)\hat{j} + (a+1)\hat{k}$$

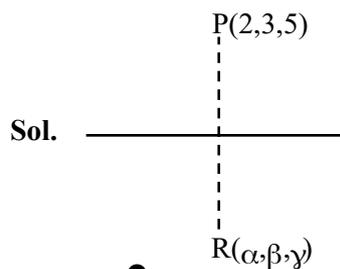
$$a - k - a - k = 0 \Rightarrow k = 0$$

 As, $PQ \perp PR$

$$(a-r)(a-k) + (a-r)(a+k) + (a-1)(a+1) = 0 \quad a$$

$$= 1 \text{ or } -1$$

$$12a^2 = 12$$

14. Ans. (2)

Sol.

$$\bullet \bullet \because PR \perp (2, 3, 4)$$

$$\bullet \bullet \therefore PR \cdot (2, 3, 4) = 0$$

$$(\alpha - 2, \beta - 3, \gamma - 5) \cdot (2, 3, 4) = 0$$

$$\Rightarrow 2\alpha + 3\beta - 4\gamma = 4 + 9 + 20 = 33$$

15. Ans. (3)

$$\text{Sol. } L_2 = \frac{x+4}{3} = \frac{y-4}{2} = \frac{z-3}{0}$$

$$\bullet \bullet \bullet \bullet \therefore \text{S.D} = \frac{\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ 2 & -3 & 2 \\ 3 & 2 & 0 \end{vmatrix}}{|n_1 \cdot n_2|}$$

$$\bullet \bullet \bullet \bullet = \frac{\begin{vmatrix} 5 & -5 & -7 \\ 2 & -3 & 2 \\ 3 & 2 & 0 \end{vmatrix}}{|n_1 \cdot n_2|}$$

$$= \frac{141}{|-4\hat{i} + 6\hat{j} + 13\hat{k}|}$$

$$= \frac{141}{\sqrt{16 + 36 + 169}}$$

$$= \frac{141}{\sqrt{221}}$$

16. Ans. (22)

$$\text{Sol. } \frac{x-4}{12} = \frac{x+6}{4} = \frac{z+2}{6}$$

$$\frac{x-4}{6} = \frac{y+6}{2} = \frac{z+2}{3} = 21$$

$$\left(21 \cdot \frac{6}{7} + 4, \frac{2}{7} \cdot 21 - 6, \frac{3}{7} \cdot 21 - 2 \right)$$

$$= (22, 0, 7) = (a, b, c)$$

$$\bullet \bullet \bullet \therefore \sqrt{324 + 144 + 16} = 22$$

17. Ans. (2)

Sol. Passing points of lines L_1 & L_2 are

$$(\lambda, 2, 1) \text{ \& } (\sqrt{3}, 1, 2)$$

$$\text{S.D.} = \begin{vmatrix} \sqrt{3} - \lambda & -1 & 1 \\ -2 & 1 & 1 \\ 1 & -2 & 1 \end{vmatrix}$$

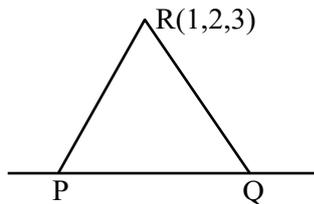
$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & 1 & 1 \\ 1 & -2 & 1 \end{vmatrix}$$

$$1 = \left| \frac{\sqrt{3} - \lambda}{\sqrt{3}} \right|$$

$$\lambda = 0, \lambda = 2\sqrt{3}$$

18. Ans. (3)

Sol.



$$P(8\lambda - 3, 2\lambda + 4, 2\lambda - 1)$$

$$PR = 6$$

$$(8\lambda - 4)^2 + (2\lambda + 2)^2 + (2\lambda - 4)^2 = 36$$

$$\lambda = 0, 1$$

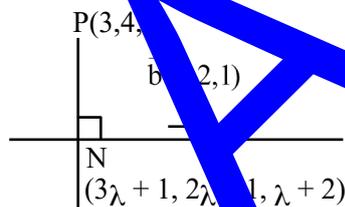
Hence $P(-3, 4, -1)$ & $Q(5, 6, 1)$

Centroid of $\Delta PQR = (\alpha, \beta, \gamma)$

$$\alpha^2 + \beta^2 + \gamma^2 = 18$$

19. Ans. (3)

Sol.



$$A(\alpha, \beta, \gamma)$$

$$PN \cdot \vec{b} = 0$$

$$3(3\lambda - 2) + 2(2\lambda - 5) + (\lambda - 7) = 0$$

$$14\lambda = 23 \Rightarrow \lambda = \frac{23}{14}$$

$$N \left(\frac{83}{14}, \frac{32}{14}, \frac{51}{14} \right)$$

$$\therefore \frac{\alpha + 3}{2} = \frac{83}{14} \Rightarrow \alpha = \frac{62}{7}$$

$$\frac{\beta + 4}{2} = \frac{32}{14} \Rightarrow \beta = \frac{4}{7}$$

$$\frac{\gamma + 9}{2} = \frac{51}{14} \Rightarrow \gamma = \frac{-12}{7}$$

$$\text{Ans. 14 } (\alpha + \beta + \gamma) = 108$$

20. Ans. (3)

$$\vec{r}_1 = (\lambda \hat{i} + 4\hat{j} + 3\hat{k}) + \alpha(2\hat{i} + 3\hat{j} + 4\hat{k})$$

$$\vec{r}_2 = (2\hat{i} + 4\hat{j} + 7\hat{k}) + \beta(2\hat{i} + 3\hat{j} + 4\hat{k})$$

$$\text{Shortest dist.} = \frac{|\vec{b} \cdot (\vec{a}_2 - \vec{a}_1)|}{|\vec{b}|} = \frac{13}{\sqrt{29}}$$

$$\frac{|(2\hat{i} + 3\hat{j} + 4\hat{k}) \cdot ((2 - \lambda)\hat{i} + 4\hat{k})|}{\sqrt{29}} = \frac{13}{\sqrt{29}}$$

$$|-8\hat{j} - \lambda\hat{k} + 12\hat{i} + 4\hat{k}| = 13$$

$$|12\hat{i} - 4\hat{j} + (4 - \lambda)\hat{k}| = 13$$

$$144 + 16\lambda^2 + (3\lambda - 4)^2 = 169$$

$$16\lambda^2 + (3\lambda - 4)^2 = 25 = \lambda \Rightarrow \lambda = 1$$

21. Ans. (1)

Sol.

$$Q(1, 6, 4)$$

$$A \left(\frac{17}{14}, \frac{48}{14}, \frac{79}{14} \right)$$

$$\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$$

$$P(\alpha, \beta, \gamma) \quad \vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$$

$$A(t, 2t + 1, 3t + 2)$$

$$\vec{QA} = (t - 1)\hat{i} + (2t - 5)\hat{j} + (3t - 2)\hat{k}$$

$$\vec{QA} \cdot \vec{b} = 0$$

$$(t - 1) + 2(2t - 5) + 3(3t - 2) = 0$$

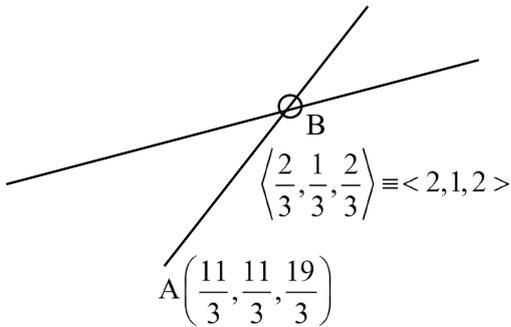
$$14t = 17$$

$$\alpha = \frac{20}{14} \quad \beta = \frac{12}{14} \quad \gamma = \frac{102}{14}$$

$$2\alpha + \beta + \gamma = \frac{154}{14} = 11$$

22. Ans. (1)

Sol. $\frac{x-1}{2-1} = \frac{y-2}{3-2} = \frac{z-3}{5-3}$
 $\Rightarrow \frac{x-1}{1} = \frac{y-2}{1} = \frac{z-3}{2} = \lambda$



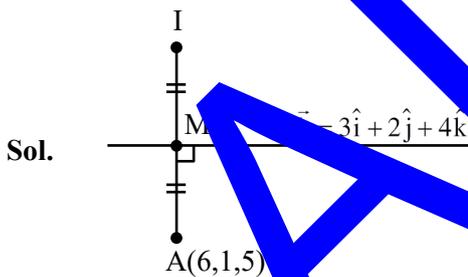
$B(1+\lambda, 2+\lambda, 3+2\lambda)$
 D.R. of AB = $\langle \frac{3\lambda-8}{3}, \frac{3\lambda-5}{3}, \frac{6\lambda-10}{3} \rangle$

$B\left(\frac{5}{3}, \frac{8}{3}, \frac{13}{3}\right)$ $\frac{3\lambda-8}{3\lambda-5} = \frac{2}{1} \Rightarrow 3\lambda-8 = 6\lambda-10$

$3\lambda = 2$
 $\lambda = \frac{2}{3}$

$AB = \frac{\sqrt{36+9+36}}{3} = \frac{9}{3} = 3$

23. Ans. (62)

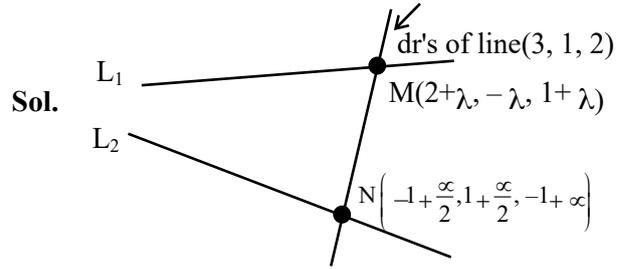


Let $M(3\lambda+1, 2\lambda, \lambda+2)$

$AM \cdot \vec{b} = 0$
 $\Rightarrow 9\lambda - 15 + 4\lambda - 2 + 16\lambda - 12 = 0$
 $\Rightarrow 29\lambda = 29$
 $\Rightarrow \lambda = 1$

$M(4, 2, 6), I = (2, 3, 7)$
 Required Distance = $\sqrt{4+9+49} = \sqrt{62}$
 Ans. 62

24. Ans. (2)



$L_1: \frac{x-2}{1} = \frac{y}{-1} = \frac{z-1}{1} = \lambda$

$L_2: \frac{x+1}{2} = \frac{y-1}{1} = \frac{z+1}{1} = \infty$

dr of line MN will be $\langle 3+\lambda-\frac{\infty}{2}, -1-\lambda-\frac{\infty}{2}, 2+\lambda-\infty \rangle$ & it will be proportional to $\langle 2, -1, 1 \rangle$

$\therefore \frac{3+\lambda-\frac{\infty}{2}}{2} = \frac{-1-\lambda-\frac{\infty}{2}}{-1} = \frac{2+\lambda-\infty}{1}$

$4\lambda + \infty = -6$ $4 + 3\lambda = 0$

$\Rightarrow \lambda = -\frac{4}{3} \text{ \& } \infty = \frac{2}{3}$

\therefore Coordinate of M will be $\langle \frac{2}{3}, \frac{4}{3}, -\frac{1}{3} \rangle$

and equation of required line will be.

$\frac{x-\frac{2}{3}}{3} = \frac{y-\frac{4}{3}}{1} = \frac{z+\frac{1}{3}}{2} = k$

So any point on this line will be

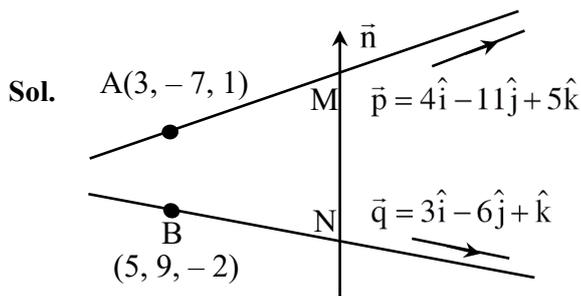
$\left(\frac{2}{3} + 3k, \frac{4}{3} + k, -\frac{1}{3} + 2k\right)$

$\therefore \frac{2}{3} + 3k = -\frac{1}{3} \Rightarrow k = -\frac{1}{3}$

\therefore Point lie on the line for

$k = -\frac{1}{3}$ is $\left(-\frac{1}{3}, 1, -1\right)$

25. Ans. (1)



$$\bullet \bullet \bullet$$

$$n = p \times q$$

$$\bullet n = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -11 & 5 \\ 3 & -6 & 1 \end{vmatrix} = 19\hat{i} + 11\hat{j} + 9\hat{k}$$

S.d. = projection of \overline{AB} on n

$$= \frac{|\overline{AB} \cdot n|}{|n|} = \frac{|(2\hat{i} + 16\hat{j} - 3\hat{k}) \cdot (19\hat{i} + 11\hat{j} + 9\hat{k})|}{\sqrt{361 + 121 + 81}}$$

$$= \frac{38 + 176 - 27}{\sqrt{563}}$$

$$\text{S.d.} = \frac{187}{\sqrt{563}}$$

26. Ans. (1)

Sol. PQ line

$$\frac{x-1}{4} = \frac{y+1}{-2} = \frac{z-3}{4}$$

$$\text{pt } (4t + 1, -2t - 1, 4t + 3)$$

$$\text{distance}^2 = 16t^2 + (-2t - 1)^2 + 16t^2 = 81$$

$$t = \pm \frac{3}{2}$$

$$\text{pt } (7, -5, 9)$$

$$\alpha^2 + \beta^2 + \gamma^2 = 155$$

option (1)

27. Ans. (48)

$$\text{Sol. } \frac{38}{3\sqrt{5}} \hat{k} = \frac{(5\hat{i} + 5\hat{j} - 9\hat{k})}{\sqrt{5}} \cdot \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 1 & -3 & 2 \end{vmatrix}$$

$$\frac{38}{3\sqrt{5}} \hat{k} = \frac{19}{\sqrt{5}}$$

$$k = \frac{19}{\sqrt{5}}$$

$$k = \frac{3}{2}$$

$$\int [x^2] = \int_0^1 x^{\sqrt{2}} dx + \int_2^{3/2} x^{3/2} dx$$

$$= \left[\frac{x^{1+\sqrt{2}}}{1+\sqrt{2}} \right]_0^1 + 2 \left[\frac{3}{2} - \sqrt{2} \right]$$

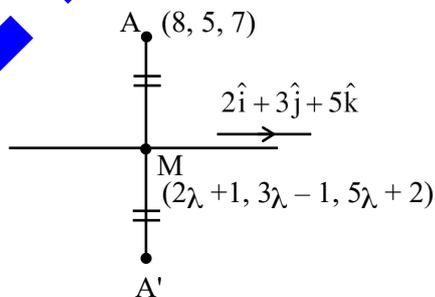
$$= 2 - \sqrt{2}$$

$$\alpha = 2$$

$$6\alpha^3 = 48$$

28. Ans. (3)

Sol.



$$\overline{AM} \cdot (2\hat{i} + 3\hat{j} + 5\hat{k}) = 0$$

$$(2\lambda - 7)(2) + (3\lambda - 6)(3) + (5\lambda - 5)(5) = 0$$

$$38\lambda = 57$$

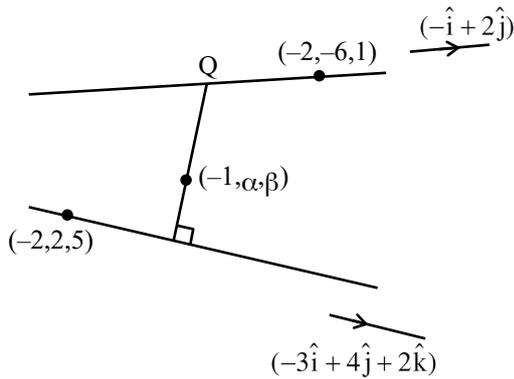
$$\lambda = \frac{3}{2}$$

$$M\left(4, \frac{7}{2}, \frac{19}{2}\right)$$

$$A'(0, 2, 12)$$

29. Ans. (25)

Sol.



$$P(-3\lambda - 2, 4\lambda + 2, 2\lambda + 5)$$

$$Q(-\alpha - 2, 2\alpha - 6, 1)$$

$$\text{DRS of PQ} = (3\lambda - \alpha, 2\alpha - 4\lambda - 8, -2\lambda - 4)$$

$$\text{DRS of PQ} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 2 & 0 \\ -3 & 4 & 2 \end{vmatrix}$$

$$= (4\hat{i} + 2\hat{j} + 2\hat{k})$$

OR

$$(2, 1, 1)$$

$$\frac{3\lambda - \alpha}{2} = \frac{2\alpha - 4\lambda - 8}{1} = \frac{2\lambda - 4}{1}$$

$$\Rightarrow \alpha = \lambda + 8, 7\lambda = \alpha - 8$$

$$\boxed{\lambda = -1} \quad \boxed{\alpha = 1}$$

$$Q : (-3, -4, 1)$$

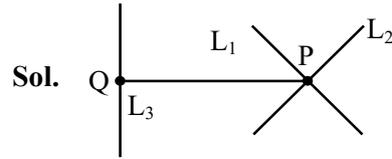
$$L_{PQ} = \frac{x+3}{2} = \frac{y+4}{1} = \frac{z-1}{1}$$

$$(-1, \alpha, \beta) \Rightarrow 1 = \frac{\alpha+4}{1} = \frac{\beta-1}{1}$$

$$\Rightarrow \alpha = -3, \beta = 2$$

$$(\alpha - \beta)^2 = 25$$

30. Ans. (3)



Sol.

$$L_1 \equiv \frac{x-2}{1} = \frac{y-4}{5} = \frac{z-2}{1} = \lambda$$

$$P(\lambda + 2, 5\lambda + 4, \lambda + 2)$$

$$L_2 \equiv \frac{x-3}{2} = \frac{y-2}{3} = \frac{z-3}{2}$$

$$P(2\alpha + 3, 3\alpha + 2, 2\alpha + 3)$$

$$\lambda + 2 = 2\alpha + 3 \quad 3\alpha + 2 = 5\lambda + 4$$

$$\lambda = 2\alpha + 1 \quad \quad \quad = 5\lambda + 2$$

$$3\alpha = 2(2\alpha + 1) + 2$$

$$3\alpha = 4\alpha + 4$$

$$\alpha = -1, \lambda = -1$$

Both satisfy (2)

$$P(-1, -1, 1)$$

$$L_3 \equiv \frac{x}{1/4} = \frac{y}{1/2} = \frac{z}{1}$$

$$L_3 \equiv \frac{y}{1} = \frac{z}{4} = k$$

Coordinates of Q(k, 2k, 4k)

$$\text{DR's of PQ} = \langle k-1, 2k+1, 4k-1 \rangle$$

PQ \perp to L₃

$$(k-1) + 2(2k+1) + 4(4k-1) = 0$$

$$k-1 + 4k+2 + 16k-4 = 0$$

$$k = \frac{1}{7}$$

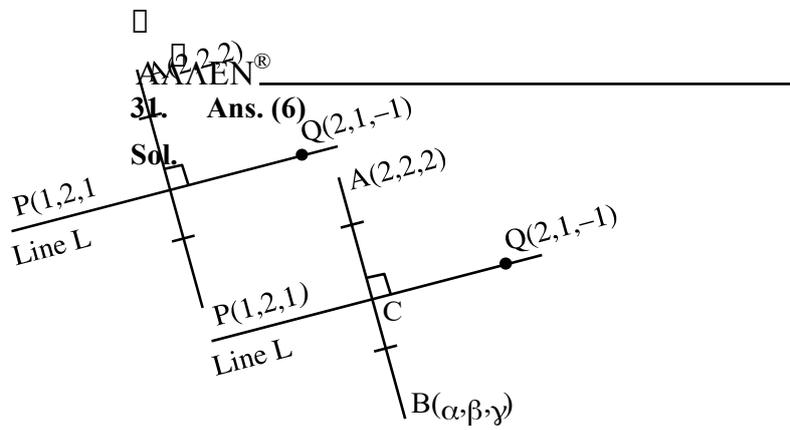
$$Q\left(\frac{1}{7}, \frac{2}{7}, \frac{4}{7}\right)$$

$$PQ = \sqrt{\left(1 - \frac{1}{7}\right)^2 + \left(-1 - \frac{2}{7}\right)^2 + \left(1 - \frac{4}{7}\right)^2}$$

$$= \sqrt{\frac{36}{49} + \frac{81}{49} + \frac{9}{49}} = \frac{\sqrt{126}}{7}$$

$$PQ = \frac{3\sqrt{4}}{7}$$

Option-3 will satisfy

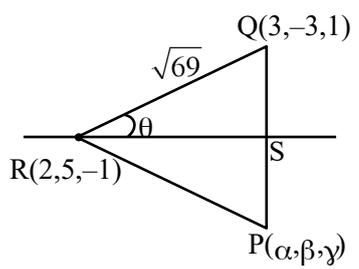


DR's of Line L $\equiv -1 : 1 : 2$
 DR's of AB $\equiv \alpha - 2 : \beta - 2 : \gamma - 2$
 $AB \perp L \Rightarrow 2 - \alpha + \beta - 2 + 2\gamma - 4 = 0$
 $2\gamma + \beta - \alpha = 4 \dots(1)$
 Let C is mid-point of AB
 $C\left(\frac{\alpha+2}{2}, \frac{\beta+2}{2}, \frac{\gamma+2}{2}\right)$
 DR's of PC $= \frac{\alpha-1}{2} : \frac{\beta-2}{2} : \frac{\gamma-1}{2}$
 $line L \parallel PC \Rightarrow \frac{-\alpha}{2} = \frac{\beta-2}{2} = \frac{\gamma}{4} = K(\text{let})$
 $\alpha = -2K$
 $\beta = 2K + 2$
 $\gamma = 4K$
 use in (1) $\Rightarrow K = \frac{1}{6}$
 value of $\alpha + \beta + 6\gamma = 24K + 2 = 6$

32. Ans. (3)

Sol. $\frac{x+6}{3} = \frac{y}{2} = \frac{z+1}{1} = \lambda \dots(1)$
 $x = 3\lambda - 6, y = 2\lambda, z = \lambda - 1$
 $\frac{x-7}{4} = \frac{y-9}{2} = \frac{z-4}{1} = \mu \dots(2)$
 $x = 4\mu + 7, y = 2\mu + 9, z = \mu + 4$
 $3\lambda - 6 = 4\mu + 7 \Rightarrow 3\lambda - 4\mu = 13 \dots(3) \times 2$
 $2\lambda = 2\mu + 9 \Rightarrow 2\lambda - 2\mu = 9 \dots(4) \times 3$
 $6\lambda - 8\mu = 26$
 $6\lambda - 9\mu = 27$
 $\underline{\quad\quad\quad} \quad \underline{\quad\quad\quad}$
 $\mu = -1$
 $\Rightarrow 3\lambda - 4(-1) = 13$
 $3\lambda = 9$
 $\lambda = 3$
 int. point (3, 6, 2); (7, 8, 9)
 $d^2 = 16 + 4 + 49 = 69$
 Ans. $d^2 + 6 = 69 + 6 = 75$

33. Ans. (4)
 Sol.



$RQ = \sqrt{1 + 64 + 4} = \sqrt{69}$
 $RQ = -8\hat{j} + 2\hat{k}$
 $RS = \hat{i} + \hat{j} - \hat{k}$
 $\cos \theta = \frac{RQ \cdot RS}{|RQ||RS|} = \frac{-8 - 2}{\sqrt{69}\sqrt{3}} = \frac{-10}{3\sqrt{23}}$
 $\cos \theta = \frac{3}{\sqrt{23}}$
 $RS = \sqrt{3}$
 $\sin \theta = \frac{\sqrt{1 - \frac{9}{23}}}{\sqrt{23}} = \frac{\sqrt{14}}{\sqrt{23}}$
 $QS = \sqrt{42}$
 $\text{Area} = \frac{1}{2} \times QS \times RS = \sqrt{42} \times 3 \times \frac{\sqrt{3}}{2}$
 $\lambda = \frac{1}{4}$
 $k^2 = 81.14 = 14k$
 $k = 81$

Ans. (43)

Sol. $\vec{a}_1 = \lambda\hat{i} + 2\hat{j} + \hat{k}$
 $\vec{a}_2 = -2\hat{i} - 5\hat{j} + 4\hat{k}$
 $\vec{p} = -3\hat{i} - \hat{j} + \hat{k}$
 $\vec{q} = 3\hat{i} + 2\hat{j} + 4\hat{k}$
 $(\lambda + 2)\hat{i} + 7\hat{j} - 3\hat{k} = \vec{a}_1 - \vec{a}_2$
 $\vec{p} \cdot \vec{q} = -6\hat{i} - 15\hat{j} + 3\hat{k}$
 $\frac{44}{\sqrt{30}} = \frac{|-6\lambda - 12 - 105 - 9|}{\sqrt{(-6)^2 + (-15)^2 + 3^2}}$
 $\frac{44}{\sqrt{30}} = \frac{|6\lambda + 126|}{3\sqrt{30}}$
 $132 = |6\lambda + 126|$
 $\lambda = 1, \lambda = -43$
 $|\lambda| = 43$

35. Ans. (4)

Sol. $\frac{2-x}{3} = \frac{3y-2}{4\lambda+1} = 4-z \dots(1)$

$$\frac{x-2}{(-3)} = \frac{y-\frac{2}{3}}{\left(\frac{4\lambda+1}{3}\right)} = \frac{z-4}{(-1)}$$

$$\frac{x+3}{3\mu} = \frac{1-2y}{6} = \frac{5-z}{7} \dots(2)$$

$$\frac{x+3}{3\mu} = \frac{y-\frac{1}{2}}{(-3)} = \frac{z-5}{(-7)}$$

Right angle \Rightarrow

$$(-3)(3\mu) + \left(\frac{4\lambda+1}{3}\right)(-3) + (-1)(-7) = 0$$

$$-9\mu - 4\lambda - 1 + 7 = 0$$

$$4\lambda + 9\mu = 6$$

36. Ans. (1)

Sol. $P(x, y, z), Q(x, y, 0); x^2 + y^2 + z^2 = y^2$

$$\overline{OQ} = x\hat{i} + y\hat{j}$$

$$\cos\theta = \frac{x}{\sqrt{x^2 + y^2}}$$

$$\cos\varphi = \frac{x}{\sqrt{x^2 + y^2 + z^2}}$$

$$\Rightarrow \sin^2\varphi = \frac{x^2 + y^2 + z^2}{x^2 + y^2 + z^2}$$

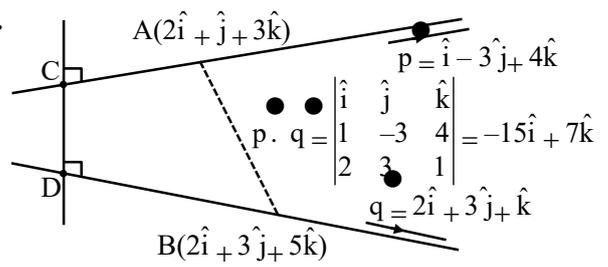
distance of P from x-axis $\sqrt{y^2 + z^2}$

$$\Rightarrow \sqrt{y^2 - x^2} \Rightarrow y\sqrt{1 - \frac{x^2}{y^2}}$$

$$= y\sqrt{1 - \cos^2\theta \sin^2\varphi}$$

37. Ans. (2)

Sol.



$$\text{Shortest distance (CD)} = \frac{|\overline{AB} \cdot \mathbf{p} \times \mathbf{q}|}{|\mathbf{p} \times \mathbf{q}|}$$

$$= \frac{|(0\hat{i} + 2\hat{j} + 2\hat{k}) \cdot (-15\hat{i} + 7\hat{j} + 9\hat{k})|}{\sqrt{55}}$$

$$= \frac{|0 + 14 + 18|}{\sqrt{55}} = \frac{32}{\sqrt{55}}$$

$$\therefore m + n = 32 + 355 = 387$$

38. Ans. (2)

Sol. $\frac{x-3}{2} = \frac{y+15}{5} = \frac{z-9}{5}$ & $\frac{x+1}{2} = \frac{y-1}{1} = \frac{z-9}{-3}$

$$\text{S.D} = \frac{|(\overline{a_2} - \overline{a_1}) \cdot (\overline{b_1} \cdot \overline{b_2})|}{|\overline{b_1} \cdot \overline{b_2}|}$$

$$a_1 = 3, -15, 9$$

$$b_1 = 2, -7, 5$$

$$a_2 = -1, 1, 9$$

$$b_2 = 2, 1, -3$$

$$a_2 - a_1 = -4, 16, 0$$

$$\overline{b_1} \cdot \overline{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -7 & 5 \\ 2 & 1 & -3 \end{vmatrix} = \hat{i}(16) - \hat{j}(-16) + \hat{k}(16)$$

$$16(\hat{i} + \hat{j} + \hat{k})$$

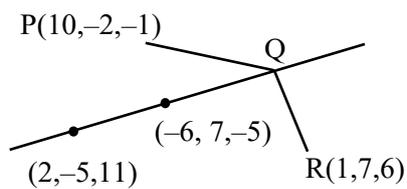
$$|\overline{b_1} \cdot \overline{b_2}| = 16\sqrt{3}$$

$$\therefore (\overline{a_2} - \overline{a_1}) \cdot (\overline{b_1} - \overline{b_2}) = 16[-4 + 16] = (16)(12)$$

$$\text{S.D.} = \frac{(16)(12)}{16\sqrt{3}} = 4\sqrt{3}$$

39. Ans. (13)

Sol.



$$\text{Line : } \frac{x+6}{-8} = \frac{y-7}{12} = \frac{z+5}{-16}$$

$$\frac{x+6}{-8} = \frac{y-7}{12} = \frac{z+5}{-16} = \lambda$$

$$Q(2\lambda - 6, 7 - 3\lambda, 4\lambda - 5)$$

$$\overline{QR} = (2\lambda - 7, -3\lambda, 4\lambda - 11)$$

$$\overline{QR} \cdot \text{dr's of line} = 0$$

$$4\lambda - 14 + 9\lambda + 16\lambda - 44 = 0$$

$$29\lambda = 58 \Rightarrow \lambda = 2$$

$$Q(-2, 1, 3)$$

$$PQ = \sqrt{144 + 9 + 16} = \sqrt{169} = 13$$

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