UNIT & DIMENSION

1. Given below are two statements :

Statement (I) : Planck's constant and angular momentum have same dimensions.

Statement (II) : Linear momentum and moment of force have same dimensions.

In the light of the above statements, choose the correct answer from the options given below :

(1) Statement I is true but Statement II is false

(2) Both Statement I and Statement II are false

(3) Both Statement I and Statement II are true

(4) Statement I is false but Statement II is true

2. The equation of state of a real gas is given by $\left(P + \dfrac{a}{V^2} \right)(V - b) = RT$, where P, V and T are pressure. volume and temperature respectively and R is the universal gas constant. The dimensions of $\dfrac{a}{b^2}$ is similar to that of :

(1) PV   (2) P   (3) RT   (4) R

3. Match List-I with List-II.

	List-I		List-II
A.	Coefficient of viscosity	I.	$[M L^2 T^{-2}]$
B.	Surface Tension	II.	$[M L^2 T^{-1}]$
C.	Angular momentum	III.	$[M L^{-1} T^{-1}]$
D.	Rotational kinetic energy	IV.	$[M L^0 T^{-2}]$

(1) A–II, B–I, C–IV, D–III

(2) A–I, B–II, C–III, D–IV

(3) A–III, B–IV, C–II, D–I

(4) A–IV, B–III, C–II, D–I

4. If mass is written as $m = k \, c^{P} \, G^{-1/2} \, h^{1/2}$ then the value of P will be : (Constants have their usual meaning with k a dimensionless constant)

(1) 1/2   (2) 1/3

(3) 2   (4) −1/3

5. A force is represented by $F = ax^2 + bt^{1/2}$

Where x = distance and t = time.

The dimensions of b^2/a are :

(1) $[ML^3T^{-3}]$ (2) $[MLT^{-2}]$

(3) $[ML^{-1}T^{-1}]$ (4) $[ML^2T^{-3}]$

6. Consider two physical quantities A and B related to each other as $E = \dfrac{B - x^2}{At}$ where E, x and t have dimensions of energy, length and time respectively. The dimension of AB is:

(1) $L^{-2} M^1 T^0$ (2) $L^2 M^1 T^{-1}$

(3) $L^{-2} M^1 T^{-1}$ (4) $L^0 M^1 T^{-1}$

7. The dimensional formula of angular impulse is :

(1) $[M L^{-2} T^{-1}]$

(2) $[M L^2 T^{-2}]$

(3) $[M L \, T^{-1}]$

(4) $[M L^2 T^{-1}]$

8. The equation of stationary wave is :

$$y = 2a \sin\left(\frac{2\pi nt}{\lambda} \right) \cos\left(\frac{2\pi x}{\lambda} \right)$$

Which of the following is NOT correct

(1) The dimensions of nt is [L]

(2) The dimensions of n is [LT^{-1}]

(3) The dimensions of n/λ is [T]

(4) The dimensions of x is [L]

9. Applying the principle of homogeneity of dimensions, determine which one is correct.

where T is time period, G is gravitational constant, M is mass, r is radius of orbit.

(1) $T^2 = \dfrac{4\pi^2 r}{GM^2}$

(2) $T^2 = 4\pi^2 r^3$

(3) $T^2 = \dfrac{4\pi^2 r^3}{GM}$

(4) $T^2 = \dfrac{4\pi^2 r^2}{GM}$

10. If G be the gravitational constant and u be the energy density then which of the following quantity have the dimension as that the $\sqrt{uG}$:

(1) Pressure gradient per unit mass

(2) Force per unit mass

(3) Gravitational potential

(4) Energy per unit mass

11. What is the dimensional formula of ab^{-1} in the equation $\left(P + \dfrac{a}{V^2}\right)(V - b) = RT$, where letters have their usual meaning.

(1) $[M^0L^3T^{-2}]$ (2) $[ML^2T^{-2}]$

(3) $[M^{-1}L^5T^3]$ (4) $[M^6L^7T^4]$

12. Match **List I** with **List II** .

	List-I		List-II
A.	Torque	I.	$[M^1L^1T^{-2}A^2]$
B.	Magnetic field	II.	$[L^2A^1]$
C.	Magnetic moment	III.	$[M^1T^{-2}A^1]$
D.	Permeability of free space	IV.	$[M^1L^2T^{-2}]$

Choose the **correct** answer from the options given below :

(1) A-I, B-III, C-II, D-IV

(2) A-IV, B-III, C-II, D-I

(3) A-III, B-I, C-II, D-IV

(4) A-IV, B-II, C-III, D-I

13. Given below are two statements :

Statement (I) : Dimensions of specific heat is $[L^2T^{-2}K^{-1}]$

Statement (II) : Dimensions of gas constant is $[M L^2T^{-1}K^{-1}]$

(1) Statement (I) is incorrect but statement (II) is correct

(2) Both statement (I) and statement (II) are incorrect

(3) Statement (I) is correct but statement (II) is incorrect

(4) Both statement (I) and statement (II) are correct

14. If $\in_0$ is the permittivity of free space and E is the electric field, then $\in_0 E^2$ has the dimensions :

(1) $[M^o L^{-2} T A]$ (2) $[M L^{-1} T^{-2}]$

(3) $[M^{-1} L^{-3} T^4 A^2]$ (4) $[M L^2 T^{-2}]$

15. The dimensional formula of latent heat is:

(1) $[M^0LT^{-2}]$

(2) $[MLT^{-2}]$

(3) $[M^0L^2T^{-2}]$

(4) $[ML^2T^{-2}]$

16. The de-Broglie wavelength associated with a particle of mass *m* and energy *E* is $h/\sqrt{2mE}$. The dimensional formula for Planck's constant is :

(1) $[ML^{-1}T^{-2}]$ (2) $[ML^2T^{-1}]$

(3) $[MLT^{-2}]$ (4) $[M^2L^2T^{-2}]$

SOLUTIONS

1. **Ans. (1)**

Sol. $[h] = ML^2T^{-1}$

$[L] = ML^2T^{-1}$

$[P] = MLT^{-1}$

$[\tau] = ML^2T^{-2}$

(Here h is Planck's constant, L is angular momentum, P is linear momentum and τ is moment of force)

2. **Ans. (2)**

Sol. $[P] = \left[\dfrac{a}{V^2}\right] \Rightarrow [a] = \left[PV^2\right]$

And $[V] = [b]$

$\dfrac{[a]}{[b^2]} = \dfrac{[PV^2]}{[V^2]} = [P]$

3. **Ans. (3)**

Sol. $F = \eta A \dfrac{dv}{dy}$

$\left[MLT^{-2}\right] = \eta \left[L^2\right]\left[T^{-1}\right]$

$\eta = \left[ML^{-1}T^{-1}\right]$

$S.T = \dfrac{F}{\ell} = \dfrac{\left[MLT^{-2}\right]}{[L]} = \left[ML^0T^{-2}\right]$

$L = mvr = \left[ML^2T^{-1}\right]$

$K.E = \dfrac{1}{2}I\omega^2 = \left[ML^2T^{-2}\right]$

4. **Ans. (1)**

Sol. $m = k \, c^P \, G^{-1/2} \, h^{1/2}$

$M^1L^0T^0 = [LT^{-1}]^P [M^{-1}L^3T^{-2}]^{1/2} [ML^2T^{-1}]^{1/2}$

By comparing $P = \frac{1}{2}$

5. **Ans. (1)**

Sol. $F = ax^2 + bt^{1/2}$

$[a] = \dfrac{[F]}{[x^2]} = [M^1L^{-1}T^{-2}]$

$[b] = \dfrac{[F]}{[t^{1/2}]} = [M^1L^1T^{-5/2}]$

$\left[\dfrac{b^2}{a}\right] = \dfrac{\left[M^2L^2T^{-5}\right]}{\left[M^1L^{-1}T^{-2}\right]} = [M^1L^3T^{-3}]$

6. **Ans. (2)**

Sol. $[B] = L^2$

$A = \dfrac{x^2}{tE} = \dfrac{L^2}{TML^2T^{-2}} = \dfrac{1}{MT^{-1}}$

$[A] = M^{-1}T$

$[AB] = \left[L^2M^{-1}T^1\right]$

7. **Ans. (4)**

Sol. Angular impulse = change in angular momentum.

[Angular impulse] = [Angular momentum] = [mvr]

$= [M L^2 T^{-1}]$

8. **Ans. (3)**

Sol. Comparing the given equation with standard equation of standing $\dfrac{2\pi n}{\lambda} = \omega$ & $\dfrac{2\pi}{\lambda} = k$

$\left[\dfrac{n}{\lambda}\right] = [\omega] = T^{-1}$

$[nt] = [\lambda] = L$

$[n] = [\lambda\omega] = LT^{-1}$

$[x] = [\lambda] = L$

Hence option (3)

9. **Ans. (3)**

Sol. According to principle of homogeneity dimension of LHS should be equal to dimensions of RHS so option (3) is correct.

$T^2 = \dfrac{4\pi^2 r^3}{GM}$

$\left[T^2\right] = \dfrac{\left[L^3\right]}{\left[M^{-1}L^3T^{-2}\right][M]}$

(Dimension of G is $\left[M^{-1}L^3T^{-2}\right]$)

$\left[T^2\right] = \dfrac{\left[L^3\right]}{\left[L^3T^{-2}\right]} = \left[T^2\right]$

10. **Ans. (2)**

Sol. $[uG] = [(M^1L^{-1}T^{-2})(M^{-1}L^3T^{-2})]$

$[uG] = [M^0L^2T^{-4}]$

$[\sqrt{uG}] = [L^1T^{-2}]$

Option (2) is correct

11. **Ans. (2)**

Sol. $\because$ $[V] = [b]$

$\therefore$ Dimension of b = $[L^3]$

& $[P] = \left|\dfrac{a}{V^2}\right|$

$[a] = [PV^2] = [ML^{-1}T^{-2}][L^6]$

Dimension of a = $[ML^5T^{-2}]$

$\therefore ab^{-1} = \dfrac{[ML^5T^{-2}]}{[L^3]} = [ML^2T^{-2}]$

12. **Ans. (2)**

Sol. $[\tau] = [r \cdot F] = [ML^2T^{-2}]$

$[F] = [qVB]$

$\Rightarrow B = \left(\dfrac{F}{qV}\right) = \left[\dfrac{MLT^{-2}}{ATLT^{-1}}\right] = [MA^{-1}T^{-2}]$

$[M] = [I \times A] = [AL^2]$

$B = \dfrac{\propto_0}{4\pi}\dfrac{Idl\ \sin\theta}{r^2}$

$\Rightarrow [\propto] = \left|\dfrac{Br^2}{Idl}\right| = \left|\dfrac{MT^{-2}A^{-1} \cdot L^2}{AL}\right|$

$= [MLT^{-2}A^{-2}]$

13. **Ans. (3)**

Sol. $\Delta Q = mS\Delta T$

$s = \dfrac{\Delta Q}{m\Delta T}$

$[s] = \left|\dfrac{ML^2T^{-2}}{MK}\right|$

$[s] = [L^2\ T^{-2}\ K^{-1}]$

Statement-(I) is correct

$PV = nRT \Rightarrow R = \dfrac{PV}{nT}$

$[R] = \dfrac{[ML^{-1}T^{-2}][L^3]}{[mol][K]}$

$[R] = [ML^2T^{-2}\ mol^{-1}K^{-1}]$

Statement-II is incorrect

14. **Ans. (2)**

Sol. $E = \dfrac{KQ}{R^2}$; $E = \dfrac{Q}{4\pi\varepsilon_o R^2}$

$\varepsilon_o = \dfrac{Q}{4\pi R^2 E}$

Now, $\varepsilon_o E^2 = \dfrac{Q}{4\pi R^2 E}.E^2 = \dfrac{Q}{4\pi R^2}.E$

$[\varepsilon_o E^2] = \left|\dfrac{QE}{R^2}\right| = \dfrac{[Q][E]}{[R^2]} = \dfrac{[Q]}{[R^2]}\dfrac{[W]}{[Q][R]}$

$= \dfrac{[W]}{[R^3]} = \dfrac{ML^2T^{-2}}{L^3} = ML^{-1}T^{-2}$

15. **Ans. (3)**

Sol. Latent heat is specific heat

$\Rightarrow \dfrac{ML^2T^{-2}}{M} = M^0L^2T^{-2}$

16. **Ans. (2)**

Sol. $\lambda = \dfrac{h}{\sqrt{2mE}}$ or E = hv

$[ML^2T^{-2}] = h[T^{-1}]$

$h = [ML^2T^{-1}]$